CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/18

Module Title: Mathematics for Engineers 402

Module Code: STAT8002

School: Mechanical, Electrical and Process Engineering

Programme Title: BEng (Hons) in Biomedical Engineering
BEng (Hons) in Mechanical Engineering

Programme Code: CR_EBIOM_8
CR_EMECH_8

External Examiner(s): Prof. Michael Wallace

Internal Examiner(s): Dr Seán Lacey

Instructions: Answer **ALL** questions

Duration: 2 hours

Sitting: Autumn 2018

NB: Questions **DO NOT** carry equal marks
Towards the back of the examination paper are required formulae.

Requirements for this examination: Formulae & Tables Booklet (State Examinations Commission)
Murdoch & Barnes Statistical Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

1. Consider a bar of heat conducting material of length 10 m. The partial differential equation describing the conduction of heat through the bar is given by

$$u_t = c^2 u_{xx}.$$

The end $x = 0$ is maintained at a temperature of $5°C$ and the end $x = 10$ is maintained at a temperature of $35°C$. The initial temperature distribution is given by $u(x, 0) = f(x)$,

(a) Use the method of separation of variables to show that a form of the solution appropriate to these boundary conditions is given by

$$u(x,t) = 5 + 3x + \sum_{n=1}^{\infty} D_n \exp\left(-\frac{c^2 n^2 \pi^2 t}{100}\right) \sin\left(\frac{n\pi x}{10}\right),$$

where the D_n are constants.

[20 marks]

(b) Hence, find the solution when $f(x) = 3x$.

[10 marks]

(c) Find and sketch the steady state solution.

[5 marks]

2. A manufacturer has 14700 hours capacity in its assembly department and 6300 hours available in its packaging department. Assembly and packaging times for each of products A, B and C are given in the table below, along with the profit per unit for each product:

	A	B	C
Assembly time (hr/unit)	0.8	0.3	0.4
Packaging time (hr/unit)	0.2	0.6	0.1
Profit per unit (€'000)	3	4	1

The problem of determining production levels so as to maximise profit may be expressed as follows:

$$\begin{aligned}\text{Maximise} \quad & z = 3x_1 + 4x_2 + x_3 \\ \text{subject to } & 8x_1 + 3x_2 + 4x_3 \leq 147, \\ & 2x_1 + 6x_2 + x_3 \leq 63, \\ & x_1, x_2, x_3 \geq 0.\end{aligned}$$

(a) Use the simplex method to show that the optimal table is given below:

Basis	x_1	x_2	x_3	s_1	s_2	RHS
x_1	1	0	1/2	1/7	−1/14	33/2
x_2	0	1	0	−1/21	4/21	5
z	0	0	1/2	5/21	23/42	139/2

[12 marks]

(b) In what units are the decision variables (x_1, x_2, x_3) expressed? What is the maximum level of profit?

[2 marks]

(c) The solution above is optimal. How is this recognised from the table?

[1 mark]

(d) In what range can the profit per unit of product A vary without changing the optimal basis?

[7 marks]

(e) In what range can the hours capacity in the assembly department vary without changing the optimal basis?

[3 marks]

(f) The following constraint $x_1 + x_2 + 2x_3 \leq 20$, related to administration time, must now be taken into consideration also. Using the optimal table solution from part (a), set up the new augmented matrix and outline (but **do not solve**) the method that would be used to find the new optimal solution.

[5 marks]

3. A small petroleum company owns two refineries. Refinery 1 costs €20,000 per day to operate, and it can produce 400 barrels of high-grade oil, 300 barrels of medium-grade oil, and 200 barrels of low-grade oil each day. Refinery 2 is newer and more modern. It costs €25,000 per day to operate, and it can produce 300 barrels of high-grade oil, 400 barrels of medium-grade oil, and 500 barrels of low-grade oil each day. The company has orders totaling 25,000 barrels of high-grade oil, 27,000 barrels of medium-grade oil, and 30,000 barrels of low-grade oil. The company would like to know how many days should it run each refinery to minimise its costs and still refine enough oil to meet its orders.

   (a) Express this minimisation problem in its standard form - i.e., formulate the equations that describe the problem outlined above.

   [5 marks]

   (b) Hence, formulate the dual maximisation problem.

   [5 marks]

   (c) By applying the simplex method to the dual maximisation problem, the following solution is obtained:

Basis	y_1	y_2	y_3	s_1	s_2	RHS
y_1	1	1/2	0	1/280	$-1/700$	250/7
y_2	0	1/2	1	$-3/1400$	1/350	200/7
z	0	500	0	25	50	$1,750,000$

   Where y_1 represents the number of barrels of high-grade oil, y_2 represents the number of barrels of medium-grade oil and y_3 represents the number of barrels of low-grade oil. Interpet the table above by outlining:

   i. The minimised cost;

   [1 mark]

   ii. How many days each refinery should be operated for;

   [2 marks]

   iii. Whether or not the original production levels have been met. Will there be a deficit/surplus of barrels produced?

   [2 marks]

4. The data below resulted from a 2^3 experiment designed to study the effects of concentration of detergent (A), concentration of sodium bicarbonate (B), and concentration of sodiumcarboxymethyl cellulose (C) on cleaning ability of a solution in washing tests. A large number indicates better cleaning ability than a smaller number. Two observations were taken at each combination of factor levels.

Test	Observation
(1)	106, 92
A	198, 199
B	197, 201
AB	329, 329
C	149, 168
AC	243, 230
BC	255, 236
ABC	383, 369

(a) Calculate and comment on the estimate of the BC interaction effect.

[8 marks]

(b) Determine an estimate of the error variance.

[3 marks]

(c) What is the null and alternative hypotheses when testing the significance of the BC interaction effect?

[2 marks]

(d) Hence, test the significance of the BC interaction effect, using a 5% level of significane.

[6 marks]

(e) State the conclusion of the test (in context).

[1 mark]

Fourier Formulae

A	0	π	2π
$\sin A$	0	0	0
$\cos A$	1	-1	1

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2n\pi}{T}t\right) + b_n \sin\left(\frac{2n\pi}{T}t\right)\right]$$

$$a_0 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)dt \; ; \qquad a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos\left(\frac{2n\pi}{T}t\right) dt \; ; \qquad b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin\left(\frac{2n\pi}{T}t\right) dt$$

Standard Integrals

$y = f(x)$	$\int f(x)\,dx$
x^n $(n \neq -1)$	$\frac{x^{n+1}}{n+1}$
e^{ax}	$\frac{1}{a}e^{ax}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\cos(ax+b)$	$\frac{1}{a}\sin(ax+b)$
$\sin(ax+b)$	$-\frac{1}{a}\cos(ax+b)$

$$\int x \sin ax\, dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2}; \quad \int x\cos ax\, dx = \frac{x\sin ax}{a} + \frac{\cos ax}{a^2}$$

Standard Derivatives

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$u \cdot v$	$v\frac{du}{dx} + u\frac{dv}{dx}$

2^k design, n replicates.

Effect estimate given by $\frac{(Contrast)}{n.2^{k-1}}$

Effect SS given by $\frac{(Contrast)^2}{n.2^k}$

$V(\text{effect estimate}) = \frac{\sigma_e{}^2}{n.2^{k-2}}$, where $\sigma_e{}^2$is the error variance .

$$SS_T = \sum (X_{ij} - \bar{X})^2,$$
$$SS_B = n \sum (\bar{X}_j - \bar{X})^2,$$
$$SS_W = \sum (X_{ij} - \bar{X}_j)^2.$$