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CIT Semester 1 Examinations 2018/19

Note to Candidates:	Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.
Module Title:	**Mathematics for Science 2.1**
Module Code:	**MATH6037**
Programme Title(s):	BSc App Physics & Instrum Y2
Block Code(s):	**SPHYS_7_Y2**
External Examiner(s):	**Prof. Brien Nolan**
Internal Examiner(s):	Dr. Michael Brennan
Instructions:	Answer Q1 (compulsory) and any other two questions. Q1 is worth 50 marks and all other questions are worth 25 marks. A table of Laplace transforms are at the end of the exam paper.
Duration:	2 Hours
Required Items:	Calculator, Log/Formulae Tables

Q1. (a) (i) Find $\dfrac{\partial f}{\partial y}$ where $f(x, y) = y^3 \sin(4x^2 y)$.

(ii) Find $\dfrac{\partial f}{\partial x}(0, 1)$ where $f(x, y) = e^{-2x+y}$.

(10 marks)

(b) Find the total differential df in terms of the differentials dw, dl and $d\theta$ given

$$f = \frac{2\pi w^3 \theta^4}{l^2}.$$

(8 marks)

(c) Use *partial fractions* to decompose

$$\frac{5x^2 - 8x + 11}{(x-2)(x^2+1)}$$

(12 marks)

(d) Find the Laplace transform of the following functions.

(i) $\mathcal{L}\{(2t+1)^2\}$

(ii) $\mathcal{L}\{2\sin(4t)\cos(2t)\}$

(10 marks)

(e) Solve using *only* the *Laplace Transform Method* for $y = y(t)$,

$$\frac{dy}{dt} - 3y = 10e^{-2t}, \qquad y(0) = 0.$$

(10 marks)

Q2. (a) Determine if $u = e^{-3y}\sin(2x)$ is a solution of Laplaces's equation

$$u_{xx} + u_{yy} = 0.$$

(8 marks)

(b) Use only the *definition* of Laplace transform to derive the Laplace transform of 3 (i.e. $\mathcal{L}\{3\}$).

(6 marks)

(c) Find the inverse Laplace transform using the *Cover Up Method*

$$\mathcal{L}^{-1}\left\{\frac{2s+1}{s(s-1)(s+4)}\right\}$$

(11 marks)

Q3. (a) Use *integration by parts* to determine

$$\int xe^{3x}\,dx.$$

(8 marks)

(b) The differential equation governing the displacement $y(t)$ of a damped oscillator is given by

$$y''(t) + y'(t) + 9.25y(t) = 0.$$

(i) Solve the differential equation using the *Laplace Transform Method*, given that $y(0) = 0$ and $y'(0) = 15$.

(ii) Determine the period of the oscillation and the duration of the oscillations.

(iii) Draw a rough sketch of $x(t)$ to illustrate your solution, labelling the axes appropriately.

(17 marks)

Q4. (a) The power consumed in an electrical resistor is given by

$$P = \frac{E^2}{R} \text{ watts}$$

where E is the voltage drop across the resistor in volts and R the resistance of the resistor in ohms.

(i) Use partial derivatives and differentials to express dP in terms of the differentials dE and dR.

(ii) Hence determine the approximate percentage change in power P when E increases by 6% and R decreases by 0.08%.

(9 marks)

(b) Find the inverse Laplace transform

$$\mathcal{L}^{-1}\left\{\frac{2s}{(s-4)^3}\right\}$$

(8 marks)

(c) Use the *Trapezoidal Rule* with $n = 5$, to approximate the integral

$$\int_1^2 \frac{1}{x}\, dx.$$

Show all your work. Round your answers to four decimal places.

(8 marks)

Laplace Transform Formulae

$$\mathcal{L}\{f(t)\} \equiv F(s) = \int_0^\infty f(t)e^{-st}\,dt = \int_0^\infty e^{-st} f(t)\,dt \qquad \textit{Definition}$$

$$\mathcal{L}\{Af(t) + Bg(t)\} = AF(s) + BG(s), \quad A, B \text{ are constants} \qquad \textit{Linearity}$$

$$\mathcal{L}\{\frac{d}{dt}f(t)\} = \mathcal{L}\{\dot{f}(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{\frac{d^2}{dt^2}f(t)\} = \mathcal{L}\{\ddot{f}(t)\} = s^2F(s) - sf(0) - \dot{f}(0)$$

$$\mathcal{L}\{f(t)e^{at}\} = \mathcal{L}\{f(t)\}|_{s\to s-a} = F(s)|_{s\to s-a} \qquad \textit{First Translation Theorem}$$

$$\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{f(t)\} = e^{-as}F(s) \qquad \textit{Second Translation Theorem}$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \qquad n = 0, 1, 2, 3, \ldots$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \qquad a \text{ is a constant}$$

$$\mathcal{L}\left\{\int_0^t f(w)\,dw\right\} = \frac{F(s)}{s}$$

$$\mathcal{L}\{\delta(t-a)\} = e^{-as}$$

$$\mathcal{L}\{\mathcal{U}(t-a)\} = \frac{e^{-as}}{s} \qquad a > 0,\ a \text{ is a positive constant}$$

$$\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2} \qquad k \text{ is a constant}$$

$$\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}$$

$$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2-k^2}$$

$$\mathcal{L}\{\cosh kt\} = \frac{s}{s^2-k^2}$$