Silence Please

Do Not turn over this page until advised to by the Invigilator

CIT Semester 1 Examinations 2018/19

Note to Candidates:	Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.
Module Title:	**Technological Mathematics 220**
Module Code:	**MATH6041**
Programme Title(s):	BEng Hons Electronic Eng Y2 BEng Electronic Engineering Y2 BSc Hons Instrument Eng Y2 BEng Electrical Engineering Y2 BEng Hons Electrical Eng Y2
Block Code(s):	**EELES_8_Y2** **EELXE_7_Y2** **SINEN_8_Y2** **EELEC_7_Y2** **EEPSY_8_Y2**
External Examiner(s):	Dr. James Cruickshank
Internal Examiner(s):	Ms. Jacqueline English
Instructions:	Answer 3 Questions. ALL questions carry equal marks (Total Marks =90)
Duration:	2 Hours
Required Items:	Calculator, Log/Formulae Tables

Q1. (a) The capacitance of a sample of 200 capacitors was measured as part of a quality assurance project. The results are tabulated as follows:

Capacitance (μF)	**Number of Capacitors**
48.0 ---- 48.5	6
48.5 ---- 49.0	25
49.0 ---- 49.5	82
49.5 ---- 50.0	61
50.0 ---- 50.5	18
50.5 ---- 51.0	5
51.0 ---- 51.5	3

(i) Calculate the mean ($\overline{x}$) and standard deviation (s) of the sample correct to 3 decimal places. [10]

(ii) Establish a cumulative frequency table and hence plot the cumulative frequency curve for this data set. [7]

(iii) Use your graph to estimate the percentage of capacitors whose capacitance is in the range $\overline{x} \pm 1.5s$. [5]

Q1 (b) Determine the five number summary for the data set below. Create a boxplot for this data. Comment on the result.

33, 35, 46, 68, 52, 46, 68, 62, 68, 69, 53, 35, 47, 68 [8]

Q2 (a) A curve is described in parametric form by

$$x = 4 - t^2 \ , \ y = t^2 + 4t$$

(i) Find $\frac{dy}{dx}$ for the curve at the point $(3,-3)$.

(ii) Find $\frac{d^2y}{dx^2}$ for the curve at this point also. [10]

Q2 (b) Air is being pumped into a spherical balloon so that its volume is increasing at a rate of $120\ cm^3/s$. How fast is the radius of the balloon increasing when the diameter is 50 cm? [10]

Q2 (c) Verify that

$$u(x, y) = e^x \sin y$$

satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$ [10]

Q3 (a) Given a curve described by the equation

$$x^2 y + y^4 - 2x = 4$$

(i) find $\frac{dy}{dx}$.

(ii) determine the equation of the tangent to this curve at the point (-1,1).

[10]

Q3 (b) Show that the equation $\ln(x) + x^2 = 3x$ has a root in the interval x = 2 to x = 3. Use three iterations at most of Newton-Raphson's method to find this root correct to two decimal places. [10]

Q3 (c) The Q factor in a resonant circuit is given by $G = \frac{R^5 T}{L^2}$

Use differentials to determine the percentage change in G when R is decreased by 2.25%, T is increased by 1.5% and L is decreased by 2.1%. [10]

Q4. (a) Evaluate each of the following integrals:

(i) $\int_0^2 te^{6t} dt$ (ii) $\int \frac{1}{2x^2 - 12x + 26} dx$

(iii) $\int \frac{40}{x^2 + 2x} dx$

[17]

Q4. (b) (i) Plot the graph $y = e^{-x}$ for values of x in the range [0, 2].

(ii) Determine the area bounded by the curve $y = e^{-x}$, the x axis and the ordinates $x = 0$ and $x = 2$ by integration.

(iii) Find the mean and the root mean square (r.m.s.) values over the same range. [13]

Statistical Formulae

Arithmetic Mean:

$\bar{x} = \dfrac{\sum x}{n}$ For a frequency distribution, $\bar{x} = \dfrac{\sum fx}{\sum f}$

Using an **assumed mean** a, where $d = x - a$

$$\bar{x} = a + \frac{\sum fd}{\sum f}$$

or $\bar{x} = a + C\dfrac{\sum f(\frac{d}{c})}{\sum f}$ where $d = x - a$ is measured in units of C

Standard Deviation:

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

For a frequency distribution $s = \sqrt{\dfrac{\sum fx^2}{\sum f} - (\bar{x})^2}$ or $s = \sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}}$

Using an **assumed mean** a, where $d = x - a$

Standard deviation $s = \sqrt{\dfrac{\sum f.d^2}{\sum f} - \left(\dfrac{\sum f.d}{\sum f}\right)^2}$

or standard deviation, $s = C\sqrt{\dfrac{\sum f.(\frac{d}{c})^2}{\sum f} - \left(\dfrac{\sum f.(\frac{d}{c})}{\sum f}\right)^2}$ where $d = x - a$ is measured in units of C