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CIT Semester 1 Examinations 2018/19

Note to Candidates:	Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.
Module Title:	**Maths for Computer Science**
Module Code:	**MATH6055**
Programme Title(s):	BSc Software Development Y1 BSc Hons Computer Systems Y1 BSc (Hons) IT Management Y1 BSc Information Technology Y1 BSc (Hons) Software Devel Y1 BSc (Hons) Web Development Y1 HC Software Dev Y1 ACCS
Block Code(s):	**KCOMP_7_Y1** **KDNET_8_Y1** **KITMN_8_Y1** **KITSP_7_Y1** **KSDEV_8_Y1** **KWEBD_8_Y1** **KCOME_6_Y1**
External Examiner(s):	Dr. James Cruickshank
Internal Examiner(s):	Dr. Marie Nicholson, Dr. J.P. Mc Carthy, Mr. Adrian O Connor, Dr. Michael Brennan, Dr. Mark Hartnett, Mr. Ali Naeem
Instructions:	Answer all 4 questions.
Duration:	2 Hours
Required Items:	Calculator

1. (a) i. Simplify the following expression as much as possible.

$$\frac{x^2 - 25}{x + 5}$$

ii. Find the solution set of the following expression for x.

$$\frac{2x}{x - 3} + 3 = \frac{6}{x - 3}$$

iii. Write the following in the form q^p, where p is a rational number.

$$q^{-4} \cdot \frac{q}{(q^{-2})^4}$$

[11 marks]

(b) Solve for x in each of the following equations:

i.

$$8 + 2^{5x} = 16$$

ii.

$$\log_2(x^2) = 3$$

[10 marks]

(c) Suppose you have an alphabet of size 36 and you want at least 4,000,000,000 distinct passwords. What is the minimum length restriction on your passwords?

[4 marks]

[25 marks]

2. (a) Simplify $\overline{\overline{A} \cup (B \cap A)}$ using only *laws of sets*. Identify the laws used in each step of your solution.

[8 marks]

(b) Given $A = \{1, 3, 5, 9, 11\}$ and the relation R on A given by:

$$R = \{(1,1), (3,3), (3,1), (3,9), (9,3), (5,5), (11,5), (5,11), (11,11)\}$$

i. Draw a graph that represents R.
ii. Is R reflexive? Give a reason for your answer.
iii. Is R symmetric? Give a reason for your answer.
iv. Is R transitive? Give a reason for your answer.

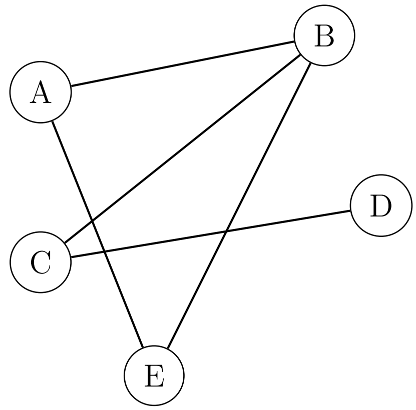
[10 marks]

(c) Let $X = P(S)$ be the power set of the set $S = \{a, b, c, d\}$.
i. Determine how many subsets of S contain exactly 3 elements.
ii. The relation $R = \{(A, B) : A, B \in X \text{ and } |A| = |B|\}$ is an equivalence relation. Write out the equivalence class $E(\{a, b, c\})$

[7 marks]

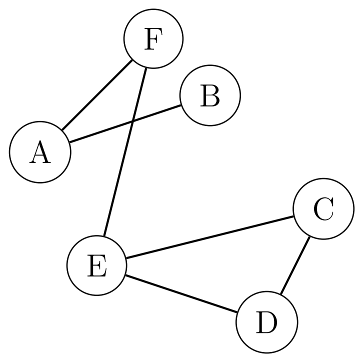
[25 marks]

3. (a) Consider the following undirected graph:



Write down the set of vertices V and the set of edges E. [5 marks]

(b) Consider a computer network given by the following (undirected) graph:



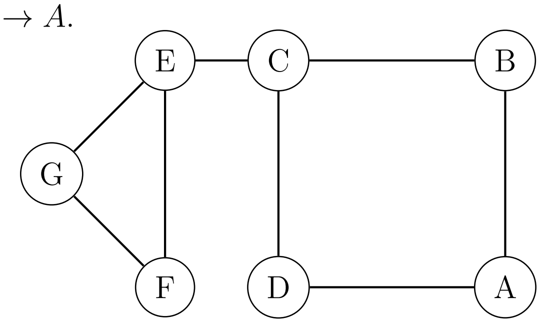
i. Give the degree of each vertex.
ii. Is the graph connected? Give a reason for your answer.
iii. Is the graph a tree? Give a reason for your answer.
iv. Does the graph have an Euler Circuit (Cycle)? If yes, find one. If not, explain your answer.
v. Does the graph have a Hamiltonian Path? If yes, find one. If not, explain your answer.

[10 marks]

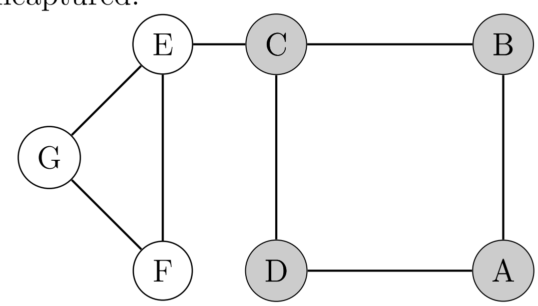
(c) i. What is the definition of a *Hamiltonian Cycle*?

ii. State Dirac's Theorem.

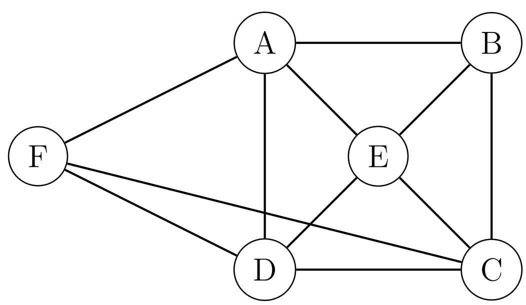
iii. A computer game consists of a player moving from vertex to vertex in a graph, in such a way that if the player travels in a cycle, all of the vertices in that cycle are 'captured', but once the cycle is completed, the game ends and no further vertices may be captured. The aim of the game is to 'capture' as many vertices as possible. For example, suppose the graph below had the player move $A \to B \to C \to D \to A$.



This forms a cycle, 'capturing' A, B, C, and D. Thereafter the game ends and E, F, and G are uncaptured:



Consider the below graph. Can <u>all</u> the vertices be captured? Justify your answer.



[10 marks]

[25 marks]

4. (a) Let $X = \{0, 1, 2, 3\}$ and let $Y = \{2, 3, 4, 5, \ldots, 10, 11\}$. Define $f : X \to Y$ as $f(x) = 3x + 2$.

   i. List the ordered pairs of the relation that define this function.

   ii. Is f onto? Give a reason for your answer.

[6 marks]

(b) Let $f : \mathbb{R} \to \mathbb{Z}$ be defined by $f(x) = \lfloor x \rfloor$ and let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = 3x$.

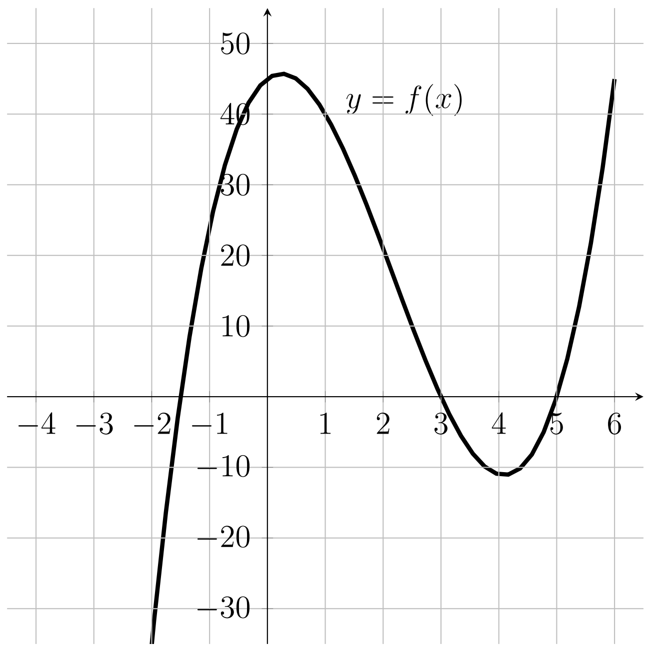
   i. Show that f is not one-to-one.

   ii. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

   iii. Is $(f \circ g)$ the same as $(g \circ f)$? Explain.

[8 marks]

(c) Use the following graph of $f(x)$ to estimate the answer of the questions below:



   i. What is $f(-1)$?

   ii. For what values of x does $f(x) = 10$?

   iii. What are the roots of $f(x)$?

   iv. Is f invertible? Give a reason for your answer.

[8 marks]

(d) List the following functions according to how fast they grow from slow to fast.

- $f(x) = 3^x + 4x$
- $g(x) = 2x^3 - 5x^2 + 3x$
- $h(x) = 6 \log_2(x)$
- $k(x) = 7x + 4$

[3 marks]

[25 marks]

Indices and Logarithms

$$a^p a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$(a^p)^q = a^{pq}$$

$$a^0 = 1$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^{\frac{1}{q}} = \sqrt[q]{a}$$

$$a^{\frac{p}{q}} = \sqrt[q]{(a)^p} = (\sqrt[q]{a})^p$$

$$(ab)^p = a^p b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

$$\log_a (xy) = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a (x^q) = q \log_a x$$

$$\log_a 1 = 0$$

$$\log_a \left(\frac{1}{x}\right) = -\log_a x$$

$$a^x = y \Leftrightarrow \log_a y = x$$

$$\log_a (a^x) = x$$

$$a^{\log_a x} = x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Sets

Name	Equality	
Double Complement Law	$\overline{(\overline{A})} = A$	
Identity Laws	$A \cap U = A$	$A \cup \varnothing = A$
Annihilation Laws	$A \cup U = U$	$A \cap \varnothing = \varnothing$
Inverse/Complement Laws	$A \cup \overline{A} = U$	$A \cap \overline{A} = \varnothing$
Idempotent Laws	$A \cup A = A$	$A \cap A = A$
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
DeMorgans Laws	$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$	$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Associative Laws	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$