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CIT Semester 1 Examinations 2018/19

Note to Candidates:	Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.
Module Title:	**Engineering Mathematics 211**
Module Code:	**MATH7006**
Programme Title(s):	BEng (Hons) Structural Eng Y2 BEng Hons Chem&Biopharm Eng Y2 BEng Hons Biomedical Eng Y2 BEng (Hons) Mechanical Eng Y2
Block Code(s):	**CSTRU_8_Y2** **ECPEN_8_Y2** **EBIOM_8_Y2** **EMECH_8_Y2**
External Examiner(s):	**Prof. Brien Nolan**
Internal Examiner(s):	Dr. Maryna Lishchynska
Instructions:	Answer ALL questions. Show all calculations in full.
Duration:	2 Hours
Required Items:	Log/Formulae Tables

Q1.

a) Given the following differential equation

$$x\frac{dy}{dx} - 3y = x^5$$

(i) Solve the equation. Give your answer in the explicit form.

(ii) Verify that your solution satisfies the differential equation.

[8 marks]

b) In a chemical reactor a substance is formed. Once the reaction has stopped, the chemical breaks down at a rate proportional to its concentration. The following differential equation models this process:

$$\frac{dC}{dt} = -kC$$

where t is time in minutes, $C(t)$ is the concentration of chemical (in parts per million, ppm) and k is the decay rate constant.

(i) Given that the decay rate constant k is 0.05 min^{-1} and the initial concentration is 10 ppm, solve this differential equation and find the concentration as a function of time, $C(t)$.

(ii) How long will it take for the concentration to drop down to 3 ppm?

[8 marks]

c) In answering parts (i) and (ii) below, you are required to use the method of undetermined coefficients. No marks will be awarded if any other method is used.

(i) Find the solution of the following initial value problem

$$\frac{d^2y}{dt^2} + 4y = 2t - 5 \qquad y(0) = y'(0) = 0$$

[11 marks]

(ii) Find the general solution of the following differential equation

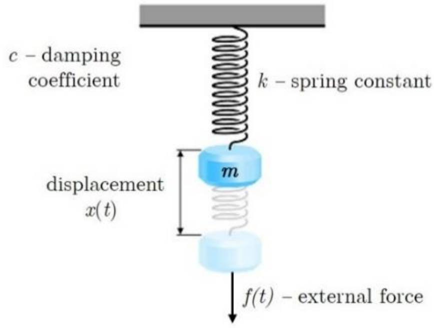
$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 6e^{-x}$$

[9 marks]

P.T.O.

d)

A mass of 3 kg is attached on a spring. A force of 2.4 N is required to stretch the spring by 0.2 m beyond its natural length. The damping coefficient is 12 and there is no external force on the mass. Initially the spring is stretched by 0.4 m beyond its natural length and then released with zero velocity.



$$m\,x'' + c\,x' + k\,x = f(t)$$

Hooke's law: $F = kx$

(i) Determine the spring constant.

(ii) Form the corresponding second order differential equation and initial conditions.

(iii) Use the method of undetermined coefficients to find the function of position of the mass $x(t)$ at any time t.

[9 marks]

P.T.O.

Q2.

a) Find the inverse Laplace transform of the following expression:

$$\frac{2s+10}{\left(s+4\right)^3}$$

[4 marks]

In parts (b) and (c) you are required to use the Laplace transforms method. No marks will be awarded if any other method is used.

b) An object, initially stored on ice, is now heating up in an oven. The temperature of the object $x(t)$ is the subject of the following initial value problem:

$$\frac{dx}{dt} + 2x = 24e^{-4t} \qquad x(0) = 0$$

Solve this problem to find $x(t)$.

[6 marks]

c) Solve the following differential equation:

$$\frac{d^2x}{dt^2} + 9x = 26e^{-2t} \qquad x(0) = x'(0) = 0$$

[10 marks]

d) An engineering system is modelled by the following integro-differential equation:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 9y + 10\int_0^t y\,du = f(t) \qquad y(0) = y'(0) = 0$$

(i) Determine the transfer function of this system and poles of the transfer function.

(ii) Investigate and state whether this physical system is stable or not.

[10 marks]

P.T.O.

Q3.

a) C is a closed curve comprising the entire perimeter of the semi-circular region defined by $x^2 + y^2 \leq 9$, $x \geq 0$. Make a sketch, show the direction of integration and evaluate the line integral

$$\oint_C xy\,dx + 2y^2 dy\,.$$

[10 marks]

b) A lamina (a thin flat plate) is modelled by the region R bounded by the line $y = 2x$ and curve $y = x^2$ between the points (0, 0) and (2, 4). Make a sketch of region R then use a double integral to find the second moment of the lamina about the y-axis.

[7 marks]

c) V is the prism with vertices (0,0,0), (1,0,0), (0,1,0), (0,0,3), (1,0,3) and (0,1,3). Evaluate

$$\iiint_V 12\,y\,z^2\,dV\,.$$

[8 marks]

Table of Laplace Transforms

$f(t)$	$F(s)$
a	$\frac{a}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin wt$	$\frac{w}{s^2+w^2}$
$\cos wt$	$\frac{s}{s^2+w^2}$
$\sinh kt$	$\frac{k}{s^2-k^2}$
$\cosh kt$	$\frac{s}{s^2-k^2}$
$e^{at}f(t)$	$F(s\text{-}a)$
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\int_0^t f(u)\,du$	$\frac{1}{s}F(s)$

$$\cosh kt = \frac{e^{kt}+e^{-kt}}{2}; \quad \sinh kt = \frac{e^{kt}-e^{-kt}}{2}$$

Useful Formulae

$$\overline{x} = \tfrac{1}{m}\iint_R x\,\rho(x,y)dA, \qquad \overline{y} = \tfrac{1}{m}\iint_R y\,\rho(x,y)dA$$

$$I_x = \iint_R y^2\,\rho(x,y)dA, \qquad I_y = \iint_R x^2\,\rho(x,y)dA$$