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CIT Semester 1 Examinations 2018/19

Note to Candidates:	Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.
Module Title:	**Mathematics for Science 3.1**
Module Code:	**MATH7010**
Programme Title(s):	BSc App Physics & Instrum Y3 BSc App Physics & Instrum Y3
Block Code(s):	**SPHYS_7_Y3** **SPHYS_7_Y3**
External Examiner(s):	Prof. Brien Nolan
Internal Examiner(s):	Ms. Mary Brennan
Instructions:	Answer all FOUR questions (worth 25 marks each). Table 1 for Q4 is on page 6. Complete the table and insert it into your exam booklet with your name clearly written on it.
Duration:	2 Hours
Required Items:	Calculator, Log/Formulae Tables

1. (a) Given the piecewise-defined function $f(t)$:

$$f(t) = \begin{cases} 3 & \text{for } 0 \leq t < 4 \\ t-4 & \text{for } 4 \leq \ t < 8 \\ -5 & \text{for } t \geq 8 \end{cases}$$

(i) Sketch the graph of $f(t)$. (4 Marks)

(ii) Write $f(t)$ in terms of unit step functions. (4 Marks)

(iii) Find the Laplace transform of $f(t)$. (7 Marks)

(b) If $f(t)$ is a periodic function with period T, then

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt.$$

Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} 2 & 0 \leq t < 3 \\ t & 3 \leq t < 6 \end{cases}$$

where $f(t+6) = f(t)$.

(10 Marks)

2. (a) Let the transfer function $G(s)$ be defined by

$$G(s) = \frac{s-4}{(s+5)(s^2-s+12)}$$

(i) Find the poles and zeros of the transfer function. (3 Marks)

(ii) Sketch the s-plane plot. (3 Marks)

(iii) Sketch the transient response of the poles. (2 Marks)

(iv) Is the system represented by this transfer function critically stable, asymptotically stable or unstable? Give a reason(s) for your answer. (2 Marks)

(b) Evaluate the following integrals:

(i) $\displaystyle\int_0^6 e^{-5t}\delta(t-9)\, dt$ (ii) $\displaystyle\int_0^\infty 8\sin(2t)\delta\left(t-\frac{\pi}{2}\right)\, dt$ (4 Marks)

(c) The equation of motion of a system is $x''(t) + 3x'(t) - 10x(t) = g(t)$ where $g(t)$ is an impulse of 3 units applied at $t = 4$. Determine an expression for the displacement of x in terms of t given that $x(0) = 1$ and $x'(0) = 2$. (11 Marks)

3. (a) Prove that the product of two odd functions is even. (5 Marks)

   (b) Find the Fourier series representation for the function $f(t)$ defined by

$$f(t) = \begin{cases} 3 & -1 \leq t < 0 \\ 5 & 0 \leq t < 1 \end{cases}$$

   and $f(t+2) = f(t)$. Hence write down the Fourier series representation of the function

$$g(t) = \begin{cases} 4 & -1 \leq t < 0 \\ 10 & 0 \leq t < 1 \end{cases}$$

   and $g(t+2) = g(t)$.

(20 Marks)

4. The set of values below contains an error.

x	-0.2	0	0.2	0.4	0.6	0.8	1.0	1.2
$f(x)$	0.27	0.25	0.27	0.38	0.43	0.57	0.75	0.97

   (i) Form a difference table for the values given in the table above, up to and including second differences. Record your answers in **Table 1** on page 6.

(6 Marks)

   (ii) Find and correct the error, ϵ, stating the necessary assumptions.

(5 Marks)

   (iii) Correct all values containing the error.

(3 Marks)

   (iv) Expand the table to determine $f(-0.4)$ and $f(1.4)$.

(4 Marks)

   (v) Use the Newton-Gregory interpolation formula to determine $f(0.9)$.

(7 Marks)

Useful formula:

- General Lagrange Interpolation Polynomial: for n a positive integer

$$\mathbf{f(x) \approx p_n(x) = \sum_{k=0}^{n} L_k(x) f_k = \sum_{k=0}^{n} \frac{l_k(x)}{l_k(x_k)} f_k}$$

where $L_k(x_k) = 1$ and 0 at the other nodes.

- Normal Equations:

$$\Sigma y = m\Sigma x + Nc$$

$$\Sigma xy = m\Sigma x^2 + c\Sigma x$$

where N is the number of ordinates.

- Newton-Gregory Interpolation (forward difference) Formula

$$\mathbf{y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + ...}$$

- Representation of a periodic function by a trigonometric Fourier series:

If the function $f(t)$ has period $T = 2L$ then

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi}{L} t + b_n \sin \frac{n\pi}{L} t)$$

with the Fourier coefficients of $f(t)$ given by the Euler formulas

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(t) dt$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(t) \cos \frac{n\pi}{L} t dt \qquad b_n = \frac{1}{L} \int_{-L}^{L} f(t) \sin \frac{n\pi}{L} t dt. \quad n = 1, 2, 3, \ldots$$

MATH7010: Table of Laplace Transforms

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a) \qquad \mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}F(s)$$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0) \qquad \mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{\delta(t-t_0)\} = e^{-st_0} \qquad \mathcal{L}\{\mathcal{U}(t-a)\} = \frac{e^{-as}}{s}$$

$\mathbf{f(t)}$	$\mathbf{F(s)}$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$te^{-\alpha t}$	$\frac{1}{(s+\alpha)^2}$
$e^{-\alpha t} - e^{-\beta t}$	$\frac{\beta-\alpha}{(s+\alpha)(s+\beta)}$
$e^{-\alpha t}\sin \omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos \omega t$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$