# Silence Please

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## CIT Semester 1 Examinations 2018/19

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| **Note to Candidates:** | Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator. |
| **Module Title:** | **Mathematics for Electronic Eng** |
| **Module Code:** | **MATH7013** |
| **Programme Title(s):** | BEng Hons Electronic Eng Y3<br>BEng Electronic Engineering Y3 |
| **Block Code(s):** | **EELES_8_Y3**     **EELXE_7_Y3** |
| **External Examiner(s):** | **Prof. Brien Nolan** |
| **Internal Examiner(s):** | Dr. Violeta Morari |
| **Instructions:** | Answer All Questions |
| **Duration:** | 2 Hours |
| **Required Items:** | Calculator, Log/Formulae Tables |

1. (a) Find $\mathcal{L}\{10t^4 + 6t^2 + e^{2t} + \frac{1}{e^{2t}}\}$.

[4 marks]

(b) Find $\mathcal{L}\{2\sin 6t \cos 4t\}$.

[3 marks]

(c) Using the results above, or otherwise, find $\mathcal{L}\{2e^{2t}\sin 6t \cos 4t\}$.

[5 marks]

(d) Find $\mathcal{L}^{-1}\left\{\frac{8}{(s+2)^4}\right\}$.

[4 marks]

(e) Find $\mathcal{L}^{-1}\left\{\frac{2-s}{(s-4)(s+2)^2}\right\}$.

[10 marks]

(f) Use Laplace transforms to solve the differential equation

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x(t) = 0, \quad x(0) = 2, \quad x'(0) = 2.$$

[8 marks]

[P.T.O]

Winter 2018

2. (a) Plot a pole-zero map for the system

$$G(s) = \frac{s(s-8)}{(s-2)(s^2+4s+5)}.$$

(i) Comment on the stability of this system.

(ii) Which, if any, of the poles of this system give rise to oscillations?

[6 marks]

(b) Consider the second order differential equation

$$\frac{d^2x}{dt^2} + x(t) = r(t) \text{ given that } x(0) = x'(0) = 0.$$

(i) Find the transfer function for the input-output system governed by this differential equation.

[4 marks]

(ii) Find the total response to the input $r(t) = e^{-3t}$, indicating the transient response and the steady state response. [A complete solution of the differential equation is required here.]

[10 marks]

(c) Sketch a graph of the function defined by

$$h(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$$

and find its Laplace Transform.

[8 marks]

(d) Find and sketch the inverse Laplace transform $g(t)$ for the function

$$G(s) = \frac{e^{-2s} - 4e^{-4s}}{s}.$$

[5 marks]

[P.T.O]

Winter 2018

3. Consider the function defined by

$$f(t) = t, \;\; -2\pi \leq t < 2\pi$$

and $f(t + 4\pi) = f(t)$ for all $t \in \mathbb{R}$.

(a) Draw a graph of the function $f(t)$ from $t = -4\pi$ to $t = 4\pi$.

[4 marks]

(b) Find the Fourier series representation of the function $f(t)$.

[17 marks]

(c) Sketch the Fourier amplitude spectrum of this waveform.

[5 marks]

(d) Find the power content $P_{av}$ of the function $f(t)$ .

[7 marks]

[P.T.O]

# Some Properties of the Laplace Transform

## Definition and Notation

$$\mathcal{L}\{f(t)\} \equiv F(s) = \int_0^\infty e^{-st} f(t) dt$$

## Linearity:

For any constants $c_1$ and $c_2$

$$\mathcal{L}\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 \mathcal{L}\{f_1(t)\} + c_2 \mathcal{L}\{f_2(t)\}$$

## The First Shift Theorem:

$$\mathcal{L}\left\{e^{-at} f(t)\right\} = F(s+a)$$

## The Second Shift Theorem:

$$\mathcal{L}\{f(t-c)u(t-c)\} = e^{-cs} F(s)$$

## Derivative:

$$\mathcal{L}\{x(t)\} = X(s)$$

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = \mathcal{L}\{x'(t)\} = sX(s) - x(0)$$

$$\mathcal{L}\left\{\frac{d^2x}{dt^2}\right\} = \mathcal{L}\{x''(t)\} = s^2 X(s) - sx(0) - x'(0)$$

## Short Table of Laplace Transforms

| $f(t)$ | $F(s) = \int_0^\infty e^{-st} f(t) dt$ |
|---|---|
| $k$ | $\frac{k}{s}$ |
| $e^{-at}$ | $\frac{1}{s+a}$ |
| $t$ | $\frac{1}{s^2}$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ |
| $\cos bt$ | $\frac{s}{s^2+b^2}$ |
| $\sin bt$ | $\frac{b}{s^2+b^2}$ |

# Fourier Series Representation

If $f(t)$ is periodic with period $T$ then

$$a_0 = \frac{2}{T}\int_0^T f(t)\ dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2\pi nt}{T}\ dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{2\pi nt}{T}\ dt$$

**Fourier Series** of the function $f$ is then given by:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n\cos\frac{2\pi nt}{T} + b_n\sin\frac{2\pi nt}{T}\right\}$$

### Average Power Content

If the function $f(t)$ is periodic with period T then the power content, $P_{av}$, of $f(t)$ is defined via

$$P_{av} = \frac{1}{T}\int_0^T [f(t)]^2\,dt.$$

It is the mean of the square of $f(t)$.

### Parseval's Theorem

If the function $f(t)$ is periodic with period T and has Fourier coefficients $a_n$ and $b_n$ so that

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n\cos\frac{2\pi nt}{T} + b_n\sin\frac{2\pi nt}{T}\right\}$$

then

$$P_{av} = \frac{1}{T}\int_0^T [f(t)]^2\,dt = \left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty}\frac{a_n^2 + b_n^2}{2}.$$

**Amplitude Phase Representation**

The amplitude-phase form of the Fourier representation of $f(t)$ is given by

$$f(t) = A_0 + \sum_{n=1}^{\infty} \Big[ A_n \cos(nt - \phi_n) \Big]$$

where $A_0 = \dfrac{a_0}{2}$, $A_n = \sqrt{a_n^2 + b_n^2}$ and $\phi_n = \tan^{-1}\left(\dfrac{b_n}{a_n}\right)$

**Discrete Frequency Spectra**

A plot of

$$A_n \text{ versus } n$$

for each $n \in N$ is an **amplitude spectrum** of $f(t)$.

A plot of

$$\phi_n \text{ versus } n$$

for each $n \in N$ is a **phase spectrum** of $f(t)$.

Between them, these spectra constitute the **discrete frequency spectra**.

$$\int t \sin at \, dt = -\frac{t}{a} \cos at + \frac{1}{a^2} \sin at$$

$$\int t \cos at \, dt = \frac{t}{a} \sin at + \frac{1}{a^2} \cos at$$