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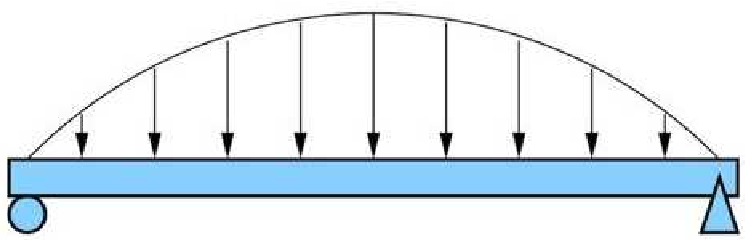
CIT Semester 1 Examinations 2018/19

Note to Candidates:	Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.
Module Title:	**Technological Mathematics 311**
Module Code:	**MATH7019**
Programme Title(s):	BEng in Civil Engineering Y3 BEng Environmental Eng Y3
Block Code(s):	**CCIVL_7_Y3** **CENVI_7_Y3**
External Examiner(s):	**Prof. Brien Nolan**
Internal Examiner(s):	Dr. J.P. Mc Carthy
Instructions:	Answer ALL Questions
Duration:	2 Hours
Required Items:	Calculator, Log/Formulae Tables

1. (a) An engineer was tasked with finding the maximum deflection due to a symmetric load on a beam of span 5 m.



She did not have a formula for the load per unit length at each point, so instead decided to make some measurements of distance from one end of the beam, x, and of the load at that point, w, and fit a curve to the measurements. She decided that the curve resembled a '$\bigcap$' parabola and so decided to fit a curve of the form

$$w = Ax^2 + Bx,$$

to the data:

x/m	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$w/\mathrm{kN\ m^{-1}}$	0.0	8.6	15.3	20.0	22.9	23.8	22.8	20.0	15.2	8.6	0.0

i. By fitting $w = Ax^2 + Bx$ to this data, find the best values of the constants A and B in the *Least Squares sense*. Use at least three significant figures for all calculations.

[HINT: $\sum wx = 392.8$, $\sum x^3 \approx 378.1$, $\sum x^2 = 96.25$, $\sum wx^2 \approx 1170$, $\sum x^4 \approx 1583$]

[7 Marks]

ii. Hence find the *location* of maximum load according your model $w = Ax^2+Bx$.

[2 Marks]

iii. Given that the load is symmetric, comment briefly on your answer to ii.

[1 Mark]

iv. Using the values of A and B found in part i., estimate the total load by evaluating

$$w_T = \int_0^5 (Ax^2 + Bx)\,dx.$$

[2 Marks]

(b) A lake has been contaminated with *E. coli* bacteria and the growth is being monitored by a team of engineers:

number of days after first detection, t	0	1	2	3	4
colony-forming units per ml, C	1.92	3.75	6.48	10.29	15.36

i. Use Lagrange Interpolation, with three suitable points, to estimate the *E. coli* level for $t = 5$.

[3 Marks]

ii. Looking at the data in its entirety, it is reasonable to postulate a relationship between C and t of the form:

$$C = a \cdot e^{bt}.$$

Write the relationship in the form $y = m \cdot x + c$; where y and x are variables, and m and c are constants.

[2 Marks]

iii. Hence use the (log-linear) Least Squares Method to find the best values of the constants a and b. Use at least three significant figures for all calculations.

[7 Marks]

iv. Use the model to estimate the *E. coli* level for $t = 5$.

[1 Mark]

2. (a) Find, in terms of EI, the maximum deflection of a light cantilever beam of span 4 m with a U.D.L. of 18 kN m^{-1} between the points $x = 1$ and $x = 2$ m and a point load of 24 kN at $x = 3$ m.

The deflection, $y(x)$, is the solution the fourth order differential equation

$$EI \cdot \frac{d^4y}{dx^4} = -w(x).$$

Equivalently, the deflection can be found by first finding the bending moment, $M(x)$, using Macauley's method or by solving the second order differential equation

$$\frac{d^2M}{dx^2} = -w(x).$$

The deflection, $y(x)$, at any point on the beam is then found by solving the differential equation

$$EI \cdot \frac{d^2y}{dx^2} = M(x).$$

[10 Marks]

(b) A light beam of span 6 m is simply supported at its endpoints. At the point $x = 3$ m there is a load of 72 kN. Between the points $x = 4$ m and $x = 6$ m there is a UDL of 72 kN m^{-1}.

i. By solving the differential equation

$$\frac{d^2M}{dx^2} = -w(x),$$

where $w(x)$ is the load per unit length, or otherwise, find the bending moment $M(x)$.

[6 Marks]

ii. Explain the true statement:

By considering the geometry of the problem, we know that the reaction at $x = 0$, R_A, is less than 108 KN.

[2 Marks]

iii. The deflection, $y(x)$, at any point on the beam is found by solving the differential equation

$$EI \cdot \frac{d^2y}{dx^2} = M(x).$$

Solve the differential equation for the deflection, $y(x)$, in terms of EI.

[5 Marks]

iv. Find, in terms of EI, the slope of the beam at $x = 0$ m.

[2 Marks]

3. A consultant is hired to analyse the manufacturing standards at a factory.

   Make all calculations correct to four significant figures.

   (a) Over a period of time she finds that 4% of produced items do not reach industry standards. The consultant decides to examine a sample of 20 items. Find the probability that:

      i. three items are substandard.

      ii. at least one item is substandard.

   [2 & 3 Marks]

   (b) Substandard items are suspected to be output according to a Poisson Distribution at an average rate of 2 per hour.

      i. Calculate the probability of the machine outputting three substandard items in a single hour.

      ii. Calculate the probability of the machine outputting no substandard items over a two hour period.

   [2 & 3 Marks]

   (c) Last year's figures suggest that the number of substandard items produced daily are normally distributed with a mean of 15 and a standard deviation of 1.2. Assuming that the distribution of substandard items has not changed, calculate the probability that the number of substandard items is

      i. greater than 18 a day.

      ii. between 13 and 17 a day.

   [2 & 3 Marks]

   (d) The consultant requested the office manager to record the number of substandard items produced for 50 working days. This study yielded a sample mean of 14.8 substandard items and a sample standard deviation of 1.3. Use these data to calculate a 95% confidence interval for the mean number of substandard items.

   [4 Marks]

   (e) Using the sample above, determine whether or not the claim that the average number of substandard items on a given day is 15 can be rejected at the 0.05 level of significance. Specify the null and alternative hypothesis.

   [6 Marks]

4. (a) Suppose that the loading on a beam of span 6 m is given by

$$w(x) = 10\cos\left(\frac{(x-3)^2}{2\pi}\right).$$

Then the shear, $V(x)$, is given by the solution of the initial value problem

$$\frac{dV}{dx} = -10\cos\left(\frac{(x-3)^2}{2\pi}\right), \quad V(0) = 24.4.$$

Use a numerical method with a step size of $h = 0.1$ to estimate the value of $V(0.2)$, the shear at $x = 20$ cm.

For those using the Three Term Taylor Method, you may use

$$\frac{d}{dx}\left(-10\cos\left(\frac{(x-3)^2}{2\pi}\right)\right) = +\frac{10(x-3)}{\pi}\sin\left(\frac{(x-3)^2}{2\pi}\right).$$

[HINT: Use RADIANS not degrees. Use at least four significant figures for intermediate calculations.]

[6 Marks]

(b) Consider the function

$$f(x) = \sqrt{1+x} = (1+x)^{1/2}.$$

i. Find the first three terms of the Maclaurin Series of $f(x)$.

[7 Marks]

ii. Hence, find correct to *eight* significant figures an approximation to $\sqrt{1.21} = \sqrt{1+0.21}$.

[2 Marks]

iii. Given that $\sqrt{1.21} = 1.1$, find, correct to one significant figure, the percentage error in this approximation.

[1 Mark]

(c) An engineer wishes to calculate the area of a circle. The area of a circle is given by

$$A = \pi r^2.$$

He measures the radius to be 9.6 m with an error of 10 cm = 0.1 m. Rather than using a good approximation to π, he uses 3.14 which has a rounding error of approximately 0.002. Use **differentials** to estimate the error in his calculation, ΔA, caused by these errors of measurement and of rounding.

Present your answer in the form

$$A = A_0 \pm \Delta A.$$

[9 Marks]

Curve Fitting

$$\sum y = m \sum x + \sum c \qquad \& \qquad \sum xy = m \sum x^2 + c \sum x$$

$$\ln(x \cdot y) = \ln x + \ln y \qquad \& \qquad \ln(x^n) = n \cdot \ln(x) \qquad \& \qquad \ln(e^x) = x$$

$$\ell(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} \cdot f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} \cdot f(x_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2)$$

Differential Equations: Beams

$$EI \cdot \frac{d^4 y}{dx^4} = -w(x)$$

$$\frac{d^2 M}{dx^2} = -w(x) \qquad \& \qquad EI \cdot \frac{d^2 y}{dx^2} = +M(x)$$

Statistics

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$\mathbb{P}[X = r] = \binom{n}{r} p^r (1 - p)^{n - r}$$

$$\mathbb{P}[X = k] = \frac{e^{-\lambda} \cdot \lambda^k}{k!}$$

$$z = \frac{X - \mu}{\sigma}$$

$$\bar{x} \pm t \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{x} \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

$$\mu \pm t \cdot \frac{\sigma}{\sqrt{n}}$$

$$\mu \pm z \cdot \frac{\sigma}{\sqrt{n}}$$

Taylor Series

$f(x)$	$f'(x)$
x^n	$n \cdot x^{n-1}$
$\frac{x^{n+1}}{n+1}$	x^n
e^{ax}	$a \cdot e^{ax}$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \, \tan x$
$\ln x$	$\frac{1}{x}$
$u \cdot v$	$u \cdot v' + v \cdot u'$
$\frac{u}{v}$	$\frac{v \cdot u' - u \cdot v'}{v^2}$
$u(v(x))$	$u'(v(x)) \cdot v'(x)$

More derivative and antiderivatives may be found in the examination tables.

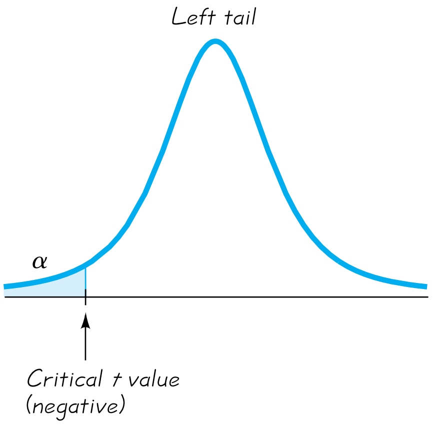
$$f(x) \approx f(a) + f'(a) \cdot (x-a) + \frac{f''(a)}{2!} \cdot (x-a)^2 + \frac{f'''(a)}{3!} \cdot (x-a)^3 + \cdots$$

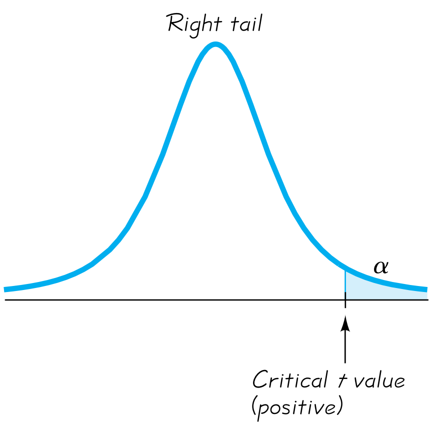
$$f(x,y) \approx f(a,b) + f_x \cdot (x-a) + f_y \cdot (y-b) + \frac{f_{xx}}{2} \cdot (x-a)^2 + \frac{f_{yy}}{2} \cdot (y-b)^2 + f_{xy} \cdot (x-a)(y-b)$$

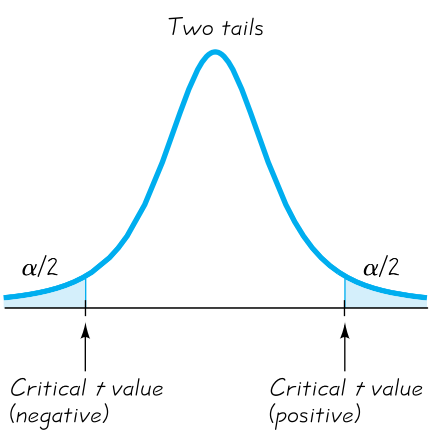
$$y_{k+1} = y_k + h \cdot y'_k + \frac{h^2}{2} y''_k$$

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	**Area in Two Tails**				
	0.01	0.02	0.05	0.10	0.20
1	**63.657**	31.821	**12.706**	6.314	3.078
2	**9.925**	6.965	**4.303**	2.920	1.886
3	**5.841**	4.541	**3.182**	2.353	1.638
4	**4.604**	3.747	**2.776**	2.132	1.533
5	**4.032**	3.365	**2.571**	2.015	1.476
6	**3.707**	3.143	**2.447**	1.943	1.440
7	**3.499**	2.998	**2.365**	1.895	1.415
8	**3.355**	2.896	**2.306**	1.860	1.397
9	**3.250**	2.821	**2.262**	1.833	1.383
10	**3.169**	2.764	**2.228**	1.812	1.372
11	**3.106**	2.718	**2.201**	1.796	1.363
12	**3.055**	2.681	**2.179**	1.782	1.356
13	**3.012**	2.650	**2.160**	1.771	1.350
14	**2.977**	2.624	**2.145**	1.761	1.345
15	**2.947**	2.602	**2.131**	1.753	1.341
16	**2.921**	2.583	**2.120**	1.746	1.337
17	**2.898**	2.567	**2.110**	1.740	1.333
18	**2.878**	2.552	**2.101**	1.734	1.330
19	**2.861**	2.539	**2.093**	1.729	1.328
20	**2.845**	2.528	**2.086**	1.725	1.325
21	**2.831**	2.518	**2.080**	1.721	1.323
22	**2.819**	2.508	**2.074**	1.717	1.321
23	**2.807**	2.500	**2.069**	1.714	1.319
24	**2.797**	2.492	**2.064**	1.711	1.318
25	**2.787**	2.485	**2.060**	1.708	1.316
26	**2.779**	2.479	**2.056**	1.706	1.315
27	**2.771**	2.473	**2.052**	1.703	1.314
28	**2.763**	2.467	**2.048**	1.701	1.313
29	**2.756**	2.462	**2.045**	1.699	1.311
30	**2.750**	2.457	**2.042**	1.697	1.310
31	**2.744**	2.453	**2.040**	1.696	1.309
32	**2.738**	2.449	**2.037**	1.694	1.309
34	**2.728**	2.441	**2.032**	1.691	1.307
36	**2.719**	2.434	**2.028**	1.688	1.306
38	**2.712**	2.429	**2.024**	1.686	1.304
40	**2.704**	2.423	**2.021**	1.684	1.303
45	**2.690**	2.412	**2.014**	1.679	1.301
50	**2.678**	2.403	**2.009**	1.676	1.299
55	**2.668**	2.396	**2.004**	1.673	1.297
60	**2.660**	2.390	**2.000**	1.671	1.296
65	**2.654**	2.385	**1.997**	1.669	1.295
70	**2.648**	2.381	**1.994**	1.667	1.294
75	**2.643**	2.377	**1.992**	1.665	1.293
80	**2.639**	2.374	**1.990**	1.664	1.292
90	**2.632**	2.368	**1.987**	1.662	1.291
100	**2.626**	2.364	**1.984**	1.660	1.290
200	**2.601**	2.345	**1.972**	1.653	1.286
300	**2.592**	2.339	**1.968**	1.650	1.284
400	**2.588**	2.336	**1.966**	1.649	1.284
500	**2.586**	2.334	**1.965**	1.648	1.283
750	**2.582**	2.331	**1.963**	1.647	1.283
1000	**2.581**	2.330	**1.962**	1.646	1.282
2000	**2.578**	2.328	**1.961**	1.646	1.282
Large	**2.576**	2.326	**1.960**	1.645	1.282







Standard Normal Probabilities

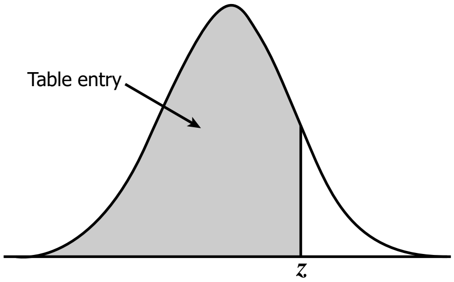


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998