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CIT Semester 1 Examinations 2018/19

Note to Candidates: Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.

Module Title: **Technological Mathematics 301**

Module Code: **MATH7020**

Programme Title(s):
BEng Mechanical Engineering Y3
BEng Hons Sustainable EnergyY3
BEng Biomedical Engineering Y3
BEng Mechanical Eng PT Y3

Block Code(s): **EMECH_7_Y3** **ESENT_8_Y3** **EBIME_7_Y3** **EMECN_7_Y3**

External Examiner(s): Prof. Brien Nolan

Internal Examiner(s): Ms. Mary Brennan

Instructions: Answer all FOUR questions (worth 25 marks each)

Duration: 2 Hours

Required Items: Calculator, Log/Formulae Tables

1. (a) Find the particular solution of the first order separable differential equation

$$2x^5\,dx + 4y\sqrt{x^6+3}\,dy = 0$$

given that $y(0) = 2$.

(7 marks)

(b) Show the integrating factor of $\dfrac{dy}{dx} = 6xy + 5x$ is e^{-3x^2}. Solve the differential equation. Hence determine the particular solution given that $y(0) = 3$.

(8 marks)

(c) Use the Method of Undetermined Coefficients to find the general solution of the non-homogeneous second order differential equation $y''(x) - 14y'(x) + 49y(x) = e^{4x} - x$.

(10 marks)

2. (a) Use Euler's Method with $h = 0.03$ to obtain an approximation to $y(4.16)$ given that y satisfies the differential equation $y' + 0.9x^2 = 1.7\sqrt{y}$, and $y(4.1) = 2$.

(10 marks)

(b) A tank containing liquid is in the form of a circular cylinder of base diameter $5m$ and of height $14m$. Let $h(t)$ be the height (in metres) of the liquid present in the tank at any time t (seconds). The tank is emptied through a circular opening at the base of diameter $0.06m$ and the velocity of the emerging liquid is given by $0.6\sqrt{2gh}$ where $g = 9.81ms^{-2}$.

(i) Show that the volume $V(t)$ of liquid in the tank at any time t is given by $6.25\pi h$.

(2 marks)

(ii) Show the differential equation representing the height $h(t)$ of liquid at time t (seconds) is

$$\frac{dh}{dt} = -3.827 \times 10^{-4}\sqrt{h}$$

(7 marks)

(iii) Solve this differential equation if initially the height of the liquid in the tank is $4m$.

(4 marks)

(iv) How long does it take (in hours) to empty the tank if initially the tank is $4m$ full?

(2 marks)

3. (a) Find the Laplace transform of

(i) $f(t) = 4t^6 - e^{0.4t} + te^{-7t}$ (ii) $f(t) = 2\sin^2(\pi t)$ (iii) $f(t) = -\frac{1}{3}t^3 e^{-1.2t}$

(10 marks)

(b) The displacement $x(t)$ (in millimetres) of a particle is described by the differential equation

$$10x'(t) + 3x(t) = 60$$

where t is the time elapsed in seconds.

(i) Use Laplace transforms to solve the equation given that the initial displacement is zero.

(10 marks)

(ii) Find the velocity and calculate the velocity initially and after 15 seconds.

(3 marks)

(iii) State the final velocity and the approximate time at which it occurs.

(2 marks)

4. (a) Find the inverse Laplace transform of

(i) $F(s) = \dfrac{2s}{(s+6)(s-8)}$ (5 marks)

(ii) $F(s) = \dfrac{s+9}{(s-7)^2 + 25}$ (5 marks)

(b) The differential equation representing free damped motion is given by

$$m\frac{d^2x}{dt^2} = -kx - \beta\frac{dx}{dt}$$

where β is a positive damping constant, k is a spring constant and m represents mass. A 10-kilogram mass is attached to a spring, stretching it 1.4 metres from its natural length. Assume a force due to air resistance which is numerically equal to 80 times the instantaneous velocity acts on the system.

(i) Show that the differential equation governing the subsequent motion is given by

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 7x = 0$$

Take $g = 9.8$ ms^{-2} and $mg = ks$ where s is the amount of elongation. (6 marks)

(ii) Use Laplace Transforms to determine the equation of motion if the mass is released from the equilibrium position with an upward velocity of 1 m/sec. (9 marks)

Short Table of Laplace Transforms

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$\mathbf{f}(\mathbf{t})$	$\mathbf{F}(\mathbf{s})$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$te^{-\alpha t}$	$\frac{1}{(s+\alpha)^2}$
$e^{-\alpha t} - e^{-\beta t}$	$\frac{\beta-\alpha}{(s+\alpha)(s+\beta)}$
$e^{-\alpha t} \sin \omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t} \cos \omega t$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$