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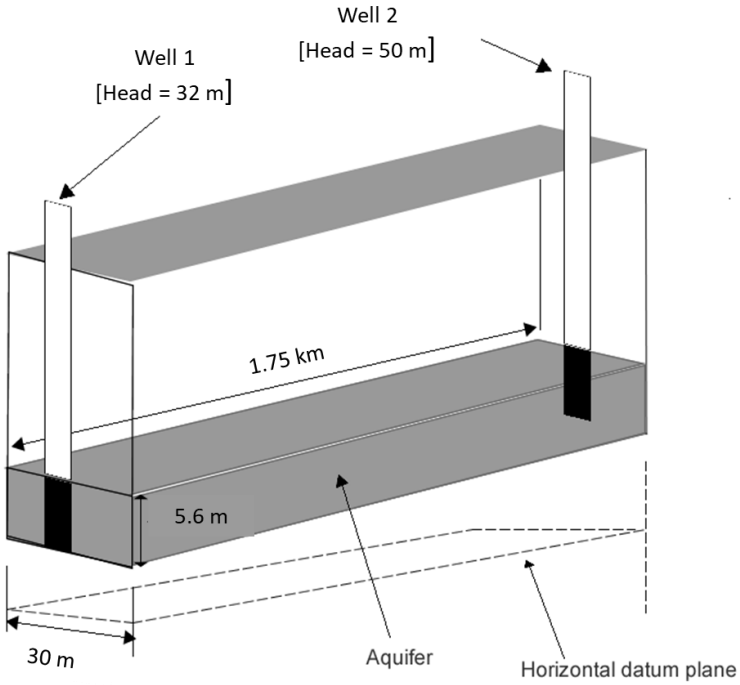
CIT Semester 1 Examinations 2018/19

Note to Candidates:	Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.
Module Title:	**Environmental Maths and Stats**
Module Code:	**MATH7025**
Programme Title(s):	BSc Hons Env Sci & Sus Tech Y3
Block Code(s):	**SESST_8_Y3**
External Examiner(s):	**Prof. Brien Nolan**
Internal Examiner(s):	Dr. Catherine Palmer
Instructions:	Answer ALL four questions.
Duration:	2 Hours
Required Items:	Calculator, Log/Formulae Tables

1.

a) Consider the aquifer shown in the diagram below. The hydraulic conductivity of the medium averages 12.3 m/day.



(i) For the given data, find the volumetric flow rate in a cross-section of this aquifer.

(ii) If the porosity of the medium is 18% find the effective fluid velocity.

(iii) Suppose a contaminant enters the groundwater upstream of the aquifer shown, at a rate of 0.3 kg/day. Find the steady state concentration of contaminant in the aquifer.

[12 marks]

b) A 120 m smokestack on level ground emits a pollutant into the atmosphere at a rate of 45 gs^{-1}. Meteorological data tells us that the mean wind speed is 19 km/hr from a north-easterly direction and that the appropriate stability class is E.

Find the mean concentration of the pollutant (in g/m^3) at a point 2 km downwind from the stack at an elevation of 10 m.

[8 marks]

2.

a) Use the method of undetermined coefficients to solve the differential equation given by:

$$y''(t) - 6y'(t) + 10y(t) = 5e^t + 2$$

subject to the initial conditions $y(0) = 1,\ y'(0) = 0$. Describe the behaviour of the solution as $t \to \infty$.

[10 marks]

b) The volume $V(t)$ (in m^3) of liquid in a tank is described by the differential equation:

$$V'(t) + 0.2V(t) = 0.1e^{-0.05t}$$

where *t* is the time elapsed in minutes.

(i) Find the volume of liquid at any time *t*, subject to the initial condition $V(0) = 0$.

(ii) What is the volume of the tank after half an hour?

(iii) At what time does the volume reach its maximum?

[10 marks]

3.

a) The concentration of PM10 in the atmosphere ($\mu g/m^3$) at Heatherton Park, Cork City was measured at four time points on a particular day and the data is shown in the table below. Note that time is measured as hours past midnight.

Hours past midnight	4	8	15	22
Concentration $\mu g m^3$	5	25	22	10

Use four-point Lagrangian interpolation to estimate the concentration of PM10 at 12 hours past midnight. [10 marks]

b) For a radioactive material, the number of remaining nuclei at time *t* can be described by the formula: $N(t) = N_0 e^{-\lambda t}$ where:

t represents time after $t = 0$ in seconds

$N(t)$ represents the number of undecayed nuclei at time *t*

N_0 represents the number of undecayed nuclei at $t = 0$

λ represents a decay constant (s^{-1})

The data in the table below shows the number of undecayed nuclei measured at different time points.

Time (sec)	0	10	50	100	200	400	800
Number of undecayed nuclei	10000	9569	7955	5632	3990	1575	241

(i) Use the Least Squares method to find the best values of N_0 and λ.

(ii) Hence estimate the number of remaining nuclei after 5 minutes.

[10 marks]

4.

a) A land owner is investigating the level of radon at a site. The levels of radon (Bq/m^3) at nine randomly chosen locations within the site are shown below.

125 155 139 142 165 128 112 135 146

(i) Calculate the mean radon level at the site and the associated standard deviation.

(ii) Set up and interpret 95% and 99% confidence intervals for the population mean. [10 marks]

b) A water treatment centre is monitoring the levels of fluoride at a certain stage of the treatment process. Over the course of the day, eight samples (each of size four) were taken and the fluoride levels (mg/L) were recorded in the table below.

Sample	1	2	3	4	5	6	7	8
Fluoride level (mg/L)	0.651	0.635	0.677	0.629	0.706	0.625	0.655	0.711
	0.710	0.649	0.610	0.673	0.716	0.612	0.672	0.510
	0.678	0.738	0.708	0.712	0.668	0.559	0.667	0.534
	0.619	0.687	0.621	0.681	0.678	0.647	0.629	0.647

Set up control charts for mean and range and test whether the process is under control.

[10 marks]

Formulae and Tables

$$C = \frac{M}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}} \qquad C = \frac{M}{4\pi t\sqrt{D_1 D_2}} e^{\left(-\frac{x^2}{4D_1 t} - \frac{y^2}{4D_2 t}\right)} \qquad C = \frac{Q}{2\pi\sigma_y \sigma_z u}\left[e^{-\frac{y^2}{2\sigma_y^2}}\right]\left[e^{-\frac{(z-H)^2}{2\sigma_z^2}} + e^{-\frac{(z+H)^2}{2\sigma_z^2}}\right]$$

Approximate values of σ_y as a function of downwind distance x in km.

Downwind Distance x (km)	σ_y (m)					
	A	B	C	D	E	F
0.1	27	19	13	8	6	4
0.2	50	36	23	15	11	8
0.4	94	67	44	29	21	14
0.7	155	112	74	48	36	24
1.0	215	155	105	68	51	34
2.0	390	295	200	130	96	64
4.0		550	370	245	180	120
7.0		890	610	400	300	200
10.0		1190	840	550	420	275
20.0		2150	1540	1000	760	500

Approximate values of σ_z as a function of downwind distance x in km.

Downwind Distance x (km)	σ_z (m)					
	A	B	C	D	E	F
0.1	14	11	7	5	4	2
0.2	29	20	14	8	6	4
0.4	72	40	26	15	11	7
0.7	215	73	43	24	17	11
1.0	455	110	61	32	21	14
2.0	1950	230	115	50	34	22
4.0		500	220	77	49	31
7.0		780	360	109	66	39
10.0		1350	510	135	79	46
20.0		2900	950	205	110	60

$$\sum y = a\sum x + bn$$
$$\sum xy = a\sum x^2 + b\sum x$$

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$f(x_0 + nh) = f_0 + n\Delta f_0 + \frac{n(n-1)}{2.1}\Delta^2 f_0 + \frac{n(n-1)n-2)}{3.2.1}\Delta^3 f_0 + \ldots$$

Control Chart Coefficients

Sample Size *n*	2	3	4	5	6	7	8	9	10
$A'_{0.025}$	1.229	0.608	0.476	0.377	0.316	0.274	0.244	0.202	0.220
$A'_{0.001}$	1.937	1.054	0.750	0.594	0.498	0.432	0.384	0.347	0.317

	For use with σ				*For use with* $\overline{R}$			
n	$D_{0.999}$	$D_{0.975}$	$D_{0.025}$	$D_{0.001}$	$D'_{0.999}$	$D'_{0.975}$	$D'_{0.025}$	$D'_{0.001}$
2	0.00	0.04	3.17	4.65	0.00	0.04	2.81	4.12
3	0.06	0.30	3.68	5.06	0.04	0.18	2.17	2.98
4	0.20	0.30	3.98	5.31	0.10	0.29	1.93	2.57
5	0.37	0.85	4.20	5.48	0.16	0.37	1.81	2.34
6	0.54	1.06	4.36	5.62	0.20	0.42	1.72	2.21
7	0.69	1.25	4.49	5.73	0.26	0.46	1.66	2.11
8	0.83	1.41	4.61	5.82	0.29	0.50	1.62	2.04
9	0.96	1.55	4.70	5.90	0.32	0.52	1.58	1.99
10	1.08	1.67	4.79	5.97	0.35	0.54	1.56	1.93
11	1.20	1.78	4.86	6.04	0.38	0.56	1.53	1.91
12	1.30	1.88	4.92	6.09	0.40	0.58	1.51	1.87

Tabular values for $\overline{x}$ and R charts

Subgroup Size	A_2	d_2	D_3	D_4
2	1.880	1.128	-----	3.268
3	1.023	1.693	-----	2.574
4	0.729	2.059	-----	2.282
5	0.577	2.326	-----	2.114
6	0.483	2.534	-----	2.004
7	0.419	2.704	0.076	1.924
8	0.373	2.847	0.136	1.864
9	0.337	2.970	0.184	1.816
10	0.308	3.078	0.223	1.777
11	0.285	3.173	0.256	1.744
12	0.266	3.258	0.283	1.717

Student *t* distribution

	20	10	5	2	1	0.2
1	3.078	6.314	12.706	31.821	63.656	318.289
2	1.886	2.920	4.303	6.965	9.925	22.328
3	1.638	2.353	3.182	4.541	5.841	10.214
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.894
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552
21	1.323	1.721	2.080	2.518	2.831	3.527
22	1.321	1.717	2.074	2.508	2.819	3.505
23	1.319	1.714	2.069	2.500	2.807	3.485
24	1.318	1.711	2.064	2.492	2.797	3.467
25	1.316	1.708	2.060	2.485	2.787	3.450
26	1.315	1.706	2.056	2.479	2.779	3.435
27	1.314	1.703	2.052	2.473	2.771	3.421
28	1.313	1.701	2.048	2.467	2.763	3.408
29	1.311	1.699	2.045	2.462	2.756	3.396
30	1.310	1.697	2.042	2.457	2.750	3.385
40	1.303	1.684	2.021	2.423	2.704	3.307
60	1.296	1.671	2.000	2.390	2.660	3.232
120	1.289	1.658	1.980	2.358	2.617	3.160