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CIT Semester 1 Examinations 2018/19

Note to Candidates:	Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.
Module Title:	**Engineering Mathematics 311**
Module Code:	**MATH8003**
Programme Title(s):	BEng (Hons) Structural Eng Y3
Block Code(s):	**CSTRU_8_Y3**
External Examiner(s):	**Prof. Brien Nolan**
Internal Examiner(s):	Dr. Maryna Lishchynska
Instructions:	Answer ALL questions. Show all calculations in full.
Duration:	2 Hours
Required Items:	

Q1.

a) Use Laplace transforms to solve the following differential equations:

(i) $$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 18\delta(t-2) \qquad y(0) = y'(0) = 0$$ **[7 marks]**

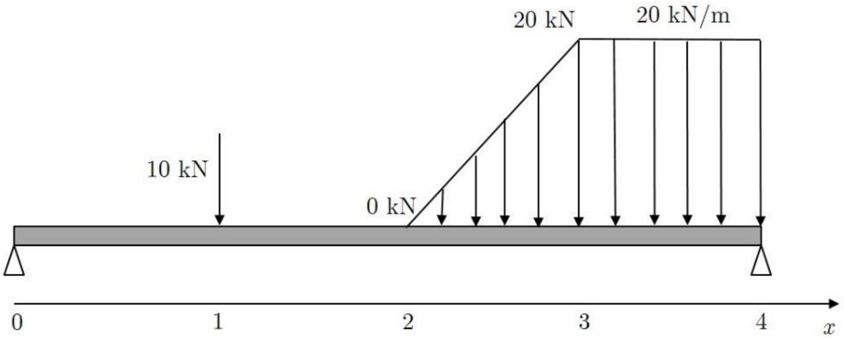
(ii) $$\frac{d^2y}{dt^2} + 4y = 12\sin 2t \qquad y(0) = y'(0) = 0$$ **[10 marks]**

b) Consider the beam under the load shown in the figure below.

(i) Plot the load function $w(x)$ as a function of x.

(ii) Express the load function $w(x)$ mathematically. Give the answer in the simplest form.

(iii) Find the Laplace transform of the load function $w(x)$ found in part (ii).



[10 marks]

c) Consider a simply supported beam of span 6 m with x-axis aligned along the length of the beam. The following differential equation relates the bending moment $M(x)$ to the beam load $w(x)$:

$$\frac{d^2M}{dx^2} = w(x).$$

There is a U.D.L. of 12 kN/m applied between the points x=2 m and x=4 m. Use Laplace transforms to solve this problem for the bending moment $M(x)$.

[8 marks]

P.T.O.

Q2.

a) Let $x(t)$, $y(t)$ and $z(t)$ be the temperatures on the three floors of an office building. Based on Newton's law of cooling the following system of differential equations models the changing temperatures in the three levels of the building:

$$\frac{dx}{dt} = 5x + 4y + 2z$$
$$\frac{dy}{dt} = 4x + 5y + 2z$$
$$\frac{dz}{dt} = 2x + 2y + 2z$$

Assume an exponential solution and hence find the general solution of this system.

[15 marks]

b) The mechanical vibrations of a structure incorporating a tuned mass damper is modelled by the following system of differential equations

$$3x_1'' = -19x_1 + 8x_2$$
$$3x_2'' = 16x_1 - 11x_2$$

where $x_1(t)$ and $x_2(t)$ represent the displacements of the main structure and the damper respectively. By assuming periodic solutions of the form $R\cos(\omega t - \alpha)$ find the general solution of the system.

[10 marks]

P.T.O.

Q3.

a) An aperiodic function $f(x)$ is defined as $f(x) = 2 - x$ where $0 \le x \le 2$.

(i) Sketch this function and its even extension.

(ii) Expand the function into half-range cosine Fourier series showing terms as far as the 5th harmonic inclusive.

[12 marks]

b) A uniform metal rod is 4 m long and aligned along the x-axis between the points $x = 0$ and $x = 4$. The temperature $u(x,t)$ at a point along the rod at any time t is found by solving the partial differential equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

The left end of the rod is maintained at 20°C and the right end is maintained at 60°C. Assume the initial temperature distribution in the rod $u(x,0)=f(x)$.

(i) Use the substitution $u(x,t) = v(x,t) + 20 + 10x$ to reduce the above problem to a homogeneous boundary value problem (BVP).

[5 marks]

(ii) Use the method of separation of variables to solve the homogeneous BVP obtained in part (i).

[14 marks]

(iii) Show that the particular solution of the original non-homogeneous BVP when $f(x) = 10x$ is

$$u(x,t) = 20 + 10x + \sum_{\substack{n=1 \\ n-odd}}^{\infty} \left(-\frac{80}{n\pi}\right) \mathbf{sin}\left(\frac{n\pi}{4}x\right) e^{-k\left(\frac{n\pi}{4}\right)^2 t}$$

[7 marks]

(iv) Find the steady-state solution and sketch it.

[2 marks]

Table of Laplace Transforms

$f(t)$	$F(s)$
a	$\frac{a}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin wt$	$\frac{w}{s^2+w^2}$
$\cos wt$	$\frac{s}{s^2+w^2}$
$\sinh kt$	$\frac{k}{s^2-k^2}$
$\cosh kt$	$\frac{s}{s^2-k^2}$
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$e^{at}f(t)$	$F(s\text{-}a)$
$\int_0^t f(u)g(t-u)du$	$F(s)G(s)$
$u(t)$	$\frac{1}{s}$
$u(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)u(t-a)$	$e^{-as}F(s)$
$\delta(t)$	1
$\delta(t-a)$	e^{-as}

Trigonometric Identities

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$\sin^2 A = \frac{1}{2}(1-\cos 2A)$

$\cos^2 A = \frac{1}{2}(1+\cos 2A)$

$\cos(-A) = \cos A; \quad \sin(-A) = -\sin A$

Fourier Formulae

A	0	π	2π
$\sin A$	0	0	0
$\cos A$	1	-1	1

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2n\pi}{T}t\right) + b_n \sin\left(\frac{2n\pi}{T}t\right)\right]$$

$$a_0 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)dt \;; \qquad a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos\left(\frac{2n\pi}{T}t\right)dt \;; \qquad b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin\left(\frac{2n\pi}{T}t\right)dt$$

Standard Integrals

$y = f(x)$	$\int f(x)\,dx$
x^n $(n \neq -1)$	$\frac{x^{n+1}}{n+1}$
e^{ax}	$\frac{1}{a}e^{ax}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\cos(ax+b)$	$\frac{1}{a}\sin(ax+b)$
$\sin(ax+b)$	$-\frac{1}{a}\cos(ax+b)$

$$\int x\sin ax\,dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2}; \quad \int x\cos ax\,dx = \frac{x\sin ax}{a} + \frac{\cos ax}{a^2}$$