Silence Please

Do Not turn over this page until advised to by the Invigilator

CIT Semester 1 Examinations 2018/19

Note to Candidates: Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.

Module Title: **Maths for Control and Quality**

Module Code: **MATH8005**

Programme Title(s):

BEng (Hons) Adv Manuf Tech Y4

BEng (Hons) Adv Manuf Tech Y4E

Cert Manuf Systems Design

Diploma Mech Eng Systems

Cert Process Plant Systems

BEng (Hons) Proc Plant Tech Y4

BEng Hons Proc Plant Tech Y4E

BSc Hons Instrument Eng Y3

Block Code(s):

EAMTE_8_Y4	**EAMTN_8_Y4**	**EMASD_8_Y1**
EMESY_8_Y1	**EPPSY_8_Y1**	**EPPTE_8_Y4**
EPPTN_8_Y4	**SINEN_8_Y3**	

External Examiner(s): Prof. Brien Nolan

Internal Examiner(s): Dr. Seán Lacey, Dr. Shane O Rourke, Joy Collins, Dr. Tadhg Creedon, Dr. Mark Hartnett, Dr. Robert Heffernan

Instructions: Answer all 4 questions.

Duration: 2 Hours

Required Items:

Question 1.

(a) Find the inverse Laplace transform of the following:

i.

$$Y(s) = \frac{7}{s} + \frac{3}{s^2} + \frac{9}{s+3}.$$

[6 marks]

ii.

$$Y(s) = \frac{1}{(s^2 + 4^2)^2}.$$

[6 marks]

(b) Solve the following initial-value problem using the method of Laplace transforms:

$$y' - 4y = u(t-3), \; y(0) = -2.$$

[6 marks]

(c) Solve the following initial-value problem using the method of Laplace transforms:

$$y'' + 4y' + 3y = \delta(t-1), \; y(0) = 0, \; y'(0) = 0.$$

[7 marks]

Question 2.

(a) Find the Z-transform of

$$(-2)^n \,.$$

[3 marks]

(b) Let $x(n) = 2^n + 4n - 1$.

i. Find the first four terms of this sequence.

[3 marks]

ii. Find the Z-transform of $x(n)$.

[6 marks]

(c) A sequence x_n is defined by the recurrence relation,

$$x_{n+1} - 3x_n = 4, \;\; x_0 = 1$$

where $n = 0, 1, 2, 3, \ldots$.

i. Find the first three terms of this sequence using the recurrence relation.

[3 marks]

ii. Use the Z-transform to find a closed-form expression for x_n.

[10 marks]

Question 3.

(a) i. The probability that a randomly selected item on a production line will be damaged before it is sent for retail is 0.004. Find the probability that out of a batch of 500 items there are more than two that will be damaged before delivery.

[4 marks]

ii. Use the Poisson approximation to the Binomial distribution to get an approximate answer to the question in part (i). How do the two results compare?

[4 marks]

(b) A company just received a shipment of 1000 exhaust muffler systems. The sampling plan for inspecting these mufflers calls for a sample size of $n = 60$ and an acceptance number $c = 1$. The contract with the muffler manufacturer calls for an acceptance quality level (AQL) of 1 defective muffler per 100 and a lot tolerance proportion defective (LTPD) of 6 mufflers per 100. Calculate

i. the operating characteristic curve for this plan; and

[13 marks]

ii. determine the producer's risk and the consumer's risk for the plan.

[4 marks]

Question 4.

(a) Suppose it is intended that there are to be at least 40 matches in a box of matches on average, but that in a random sample of 25 boxes of matches the sample mean was found to be only 39.2 matches per box with a sample standard deviation of 1.2. Does this imply that the mean number of matches per box is less than it should be? Perform a suitable hypothesis test using a 1% level of significance.

[12 marks]

(b) An engineer estimates that the average demand for electricity at a particular sub-station is 4.9 MW at 6.00pm with a standard deviation of 0.4 MW. However in a random sample of 20 days in the last year the average demand was 4.7 MW. Assuming that 0.4 MW is the standard deviation of the demand for electricity for all days, perform a suitable hypothesis test at

i. a 5% level of significance;

ii. a 1% level of significance.

to determine whether the engineer's estimate of the average demand is correct.

[13 marks]

Laplace transforms

Common Laplace transforms

$y(t) = \mathscr{L}^{-1}[Y]$	$Y(s) = \mathscr{L}[y]$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$u(t-a)$	$\frac{e^{-as}}{s}$
$\delta(t-a)$	e^{-as}

Rules for Laplace transforms

Rule for $\mathscr{L}$	Rule for $\mathscr{L}^{-1}$
$\mathscr{L}[y+w] = \mathscr{L}[y] + \mathscr{L}[w] = Y(s) + W(s)$	$\mathscr{L}^{-1}[Y+W] = \mathscr{L}^{-1}[Y] + \mathscr{L}^{-1}[W] = y(t) + w(t)$
$\mathscr{L}[\alpha y] = \alpha \mathscr{L}[y] = \alpha Y(s)$	$\mathscr{L}^{-1}[\alpha Y] = \alpha \mathscr{L}^{-1}[Y] = \alpha y(t)$
$\mathscr{L}[u(t-a)y(t-a)] = e^{-as}\mathscr{L}[y] = e^{-as}Y(s)$	$\mathscr{L}^{-1}[e^{-as}Y] = u(t-a)y(t-a)$
$\mathscr{L}[e^{at}y(t)] = Y(s-a)$	$\mathscr{L}^{-1}[Y(s-a)] = e^{at}\mathscr{L}^{-1}[Y] = e^{at}y(t)$

Laplace transforms of derivatives

$$\mathscr{L}[y'] = s\mathscr{L}[y] - y(0) = sY(s) - y(0)$$
$$\mathscr{L}[y''] = s^2\mathscr{L}[y] - sy(0) - y'(0) = s^2Y(s) - sy(0) - y'(0)$$

Z-transforms

The Z-transform of $x(n)$ is

$$\mathscr{Z}[x(n)] = \sum_{n=0}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} \frac{x(n)}{z^n}.$$

We commonly write $\mathscr{Z}[x(n)] = X(z)$.

Common Z-transforms

$x(n)$	$X(z)$
1 or $u(n)$	$\frac{z}{z-1}$
a^n	$\frac{z}{z-a}$
n	$\frac{z}{(z-1)^2}$
na^{n-1}	$\frac{z}{(z-a)^2}$
$n(n-1)$	$\frac{2z}{(z-1)^3}$
n^2	$\frac{z^2+z}{(z-1)^3}$
$\delta(n)$	1
$x(n+1)$	$zX(z) - zx(0)$
$x(n+2)$	$z^2X(z) - z^2x(0) - zx(1)$
$x(n-m)u(n-m)$	$z^{-m}X(z)$

Linearity for Z-transforms

$$\begin{aligned}
\mathscr{Z}[x(n) + y(n)] &= \mathscr{Z}[x(n)] + \mathscr{Z}[y(n)] \\
\mathscr{Z}[\alpha x(n)] &= \alpha \mathscr{Z}[x(n)] \\
\mathscr{Z}^{-1}[X(z) + Y(z)] &= \mathscr{Z}^{-1}[X(z)] + \mathscr{Z}^{-1}[Y(z)] \\
\mathscr{Z}^{-1}[\alpha X(z)] &= \alpha \mathscr{Z}^{-1}[X(z)]
\end{aligned}$$

Probability

Complement law $\mathrm{P}(\bar{A}) = 1 - \mathrm{P}(A)$

Addition law $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

Conditional probability $\mathrm{P}(B|A) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(A)}$

Multiplication law $\mathrm{P}(A \cap B) = \mathrm{P}(B|A)P(A)$

Mean of a discrete random variable

$$\mathrm{E}(X) = \sum_i x_i\,\mathrm{P}(x_i)$$

Mean of a continuous random variable

$$\mathrm{E}(X) = \int_{-\infty}^{\infty} x f(x)\ dx \text{ where } f(x) \text{ is the probability density function}$$

Variance of a random variable

$$\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$$

Binomial distribution $\mathrm{P}(X = r) = \dbinom{n}{r} p^r (1-p)^{n-r} = {}^nC_r p^r (1-p)^{n-r}$

Poisson distribution $P(X = r) = e^{-\lambda} \left(\dfrac{\lambda^r}{r!} \right)$

Acceptance sampling operating characteristic

$$L(p) = P(X \le c) = \sum_{k=0}^{c} P(X = k) = \sum_{k=0}^{c} \binom{n}{k} p^k (1-p)^{n-k} = \sum_{k=0}^{c} {}^nC_k p^k (1-p)^{n-k}$$

Sampling theory

Sample mean $\bar{x} = \dfrac{\sum x}{n}$

Sample standard deviation $s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$

Sampling distribution of the mean $\mathrm{E}(\bar{X}) = \mu \quad \mathrm{SE}(\bar{X}) = \dfrac{\sigma}{\sqrt{n}}$

Normal distribution

$$Z = \frac{x - \mu}{\sigma} \text{ where } X \sim N(\mu, \sigma)$$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \text{ where } \bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Statistical inference

Estimation / Confidence intervals

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \qquad \bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}} \qquad \bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$$

$$\frac{(n-1)s^2}{\chi^2_{\text{hi}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{lo}}}$$

$$\text{where } \chi^2_{\text{hi}} = \chi^2_{\alpha/2,n-1} \text{ and } \chi^2_{\text{lo}} = \chi^2_{1-\alpha/2,n-1}$$

Hypothesis testing

One sample tests

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \qquad z = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} \qquad t_{n-1} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$$

Two sample tests

$$z = \frac{(\bar{x_1}-\bar{x_2})-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$$

$$t_{n_1+n_2-2} = \frac{(\bar{x_1}-\bar{x_2})-(\mu_1-\mu_2)}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \text{ where } s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$$

Standard Normal Probabilities

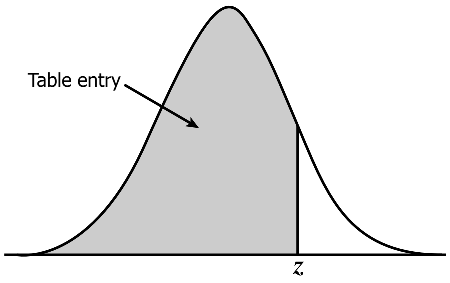


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

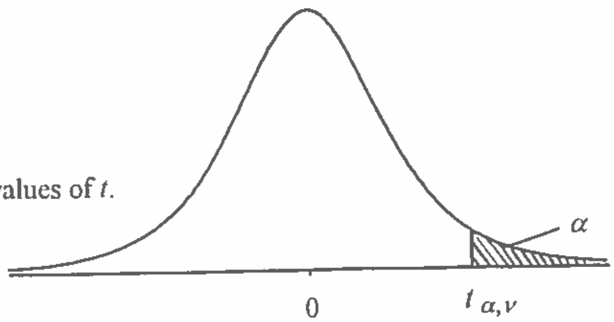
Table 7 Percentage Points of the t Distribution

The table gives the value of $t_{\alpha,\nu}$ – the 100α percentage point of the t distribution for ν degrees of freedom.

The values of t are obtained by solution of the equation:

$$\alpha = \Gamma[\tfrac{1}{2}(\nu+1)][\Gamma(\tfrac{1}{2}\nu)]^{-1}(\nu\pi)^{-1/2}\int_t^\infty (1+x^2/\nu)^{-(\nu+1)/2}\,dx$$

Note: The tabulation is for one tail only, that is, for positive values of t.
For $|t|$ the column headings for α should be doubled.



$\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
$\nu = 1$	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

This table is taken from Table III of Fisher & Yates: *Statistical Tables for Biological, Agricultural and Medical Research*, reprinted by permission of Addison Wesley Longman Ltd. Also from Table 12 of *Biometrika Tables for Statisticians*, Volume 1, by permission of Oxford University Press and the Biometrika Trustees.