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# CIT Semester 1 Examinations 2018/19

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| **Note to Candidates:** | Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator. |
| **Module Title:** | **Maths for Electrical Eng** |
| **Module Code:** | **MATH8008** |
| **Programme Title(s):** | BEng Hons Electrical Eng Y4 |
| **Block Code(s):** | **EEPSY_8_Y4** |
| **External Examiner(s):** | **Prof. Brien Nolan** |
| **Internal Examiner(s):** | Dr. Michael Brennan |
| **Instructions:** | Answer all questions. All questions carry 30 marks and so total marks for the paper is 120. A table of Laplace transforms, a table of Z-transforms and the required formulae for reliability are located at the end of the paper. |
| **Duration:** | 2 Hours |
| **Required Items:** | Calculator, Log/Formulae Tables |

Q1. (a) Use $\mathcal{L}\{g(t)\} = \int_0^\infty e^{-st} g(t)\, dt$ to prove the *Second Translation Theorem.*

$$\mathcal{L}\{f(t-a)\, \mathcal{U}(t-a)\} = e^{-as}\, \mathcal{L}\{f(t)\}$$

(8 marks)

(b) Sketch the following functions and find their Laplace transforms.

(i) $g(t) = \delta(t) + 4\delta(t-2) + 2\delta(t-5)$

(ii) $h(t) = \sin 2t\, \mathcal{U}(t - \frac{\pi}{2}))$

(12 marks)

(c) Solve the following linear second order differential equation for $y(t)$ where $t \geq 0$,

$$\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 15y = 12\delta(t-2)$$

subject to the initial conditions $y(0) = 0$ and $\dot{y}(0) = 0$.

(10 marks)

Q2. (a) The transformation matrix is given by

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

(i) Find the characteristic equation of the matrix $A$.

(4 marks)

(ii) Hence find the eigenvalues of the matrix $A$.

(5 marks)

(iii) Determine the corresponding eigenvectors of the matrix $A$

(9 marks)

(iv) Construct the modal matrix $M$ and explain how it diagonalises $A$.

(6 marks)

(b) Consider the second order differential equation given by

$$3y''(t) + 6y'(t) + 10y(t) = 10\delta(t-3)$$

Find the state-space model for this single input, single output system.

(6 marks)

Q3. (a) Consider the sequence defined by

$$a(n) = n + (-4)^n - 10\delta(n-1), \text{ for } n = 0, 1, 2, \ldots$$

where $\delta(n)$ represents the discrete delta function.

(i) Write down the first four terms of the sequence $a(n)$.

(3 marks)

(ii) Calculate the $\mathcal{Z}$-transform of the sequence $a(n)$.

(4 marks)

(iii) Write down first four terms of $a(n-2)u(n-2)$.

(4 marks)

(iv) Calculate the $\mathcal{Z}$-transform of $a(n-2)u(n-2)$.

(3 marks)

(b) Consider the second order difference equation given by:

$$y(n+2) - 3y(n+1) + 2y(n) = (-1)^n$$

where $y(0) = 0$ and $y(1) = 0$.

(i) Find the values of $y(2)$ and $y(3)$ using the recursion method.

(4 marks)

(ii) Solve the second order difference equation for $y(n)$ using the $\mathcal{Z}$-transform method.

(10 marks)

(iii) Verify your answer from (i) using the result from (ii).

(2 marks)

Q4. (a) The life of Digidell computer chips is exponentially distributed, with a mean lifetime of 2000 hours.

(i) Find the reliability function $R(t)$ for Digidell chips. Sketch a graph of $R(t)$.

(3 marks)

(ii) Find the median life of Digidell chips.

(3 marks)

(iii) What percentage of Digidell chips fail in the first 50 hours?

(2 marks)

(b) The length of life in hours of a component has a Weibull distribution with a shape parameter $\beta = 0.5$ and mean life of 5000 hours.

(i) Find the value of the scale parameter $\eta$.

(3 marks)

(ii) Find the reliability of this component for 1500 hours.

(4 marks)

(iii) Determine the failure rate function $r(t)$ for this component and calculate the rate at which components are failing at 10 and 20 hours. Comment on whether we are dealing with early or late life stage of the bathtub curve.

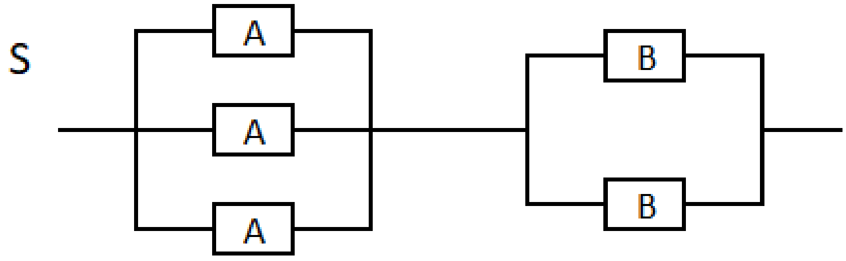
(5 marks)

(c) Find the reliability of the following system $S$ for a mission of 750 hours given that;

Component A has a Weibull lifetime with shape parameter $\beta = 2$ and scale parameter $\eta = 1500$,

Component B has exponential life with a mean lifetime of 2500 hours.

(10 marks)



| **Table of Laplace Transforms** | | **Table of $\mathcal{Z}$-Transforms** | |
|---|---|---|---|
| $f(t)$ for $t \geq 0$ | $\mathcal{L}(f) = \int_0^\infty e^{-st} f(t) dt$ | $f(n)$ | $\mathcal{Z}(f(n)) = \sum_{n=0}^{\infty} f(n) z^{-n}$ |
| $1$ | $\frac{1}{s}$ | $\delta(n-k)$ | $\frac{1}{z^k}$ |
| $e^{at}$ | $\frac{1}{s-a}$ | $u(n)$ or $1$ | $\frac{z}{z-1}$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ $(n = 0, 1, \ldots)$ | $a^n$ or $a^n u(n)$ | $\frac{z}{z-a}$ |
| $\sin(bt)$ | $\frac{b}{s^2+b^2}$ | $na^{n-1}$ | $\frac{z}{(z-a)^2}$ |
| $\cos(bt)$ | $\frac{s}{s^2+b^2}$ | $na^n$ | $\frac{az}{(z-a)^2}$ |
| $\sinh(bt)$ | $\frac{b}{s^2-b^2}$ | $n$ | $\frac{z}{(z-1)^2}$ |
| $\cosh(bt)$ | $\frac{s}{s^2-b^2}$ | $\cos(\omega n)$ | $\frac{z(z-\cos(\omega))}{z^2-2z\cos(\omega)+1}$ |
| $u(t-a)$ | $\frac{e^{-as}}{s}$ | $\sin(\omega n)$ | $\frac{z\sin(\omega)}{z^2-2z\cos(\omega)+1}$ |
| $\delta(t-a)f(t)$ | $e^{-as}f(a)$ | $f(n+1)$ | $zF(z) - zf(0)$ |
| $f'(t)$ | $s\mathcal{L}(f) - f(0)$ | $f(n+2)$ | $z^2F(z) - z^2f(0) - zf(1)$ |
| $f''(t)$ | $s^2\mathcal{L}(f) - sf(0) - f'(0)$ | $f(n+k)$ | $z^k\left(F(z) - \sum_{m=0}^{k-1} f(m)z^{-m}\right)$ |
| $e^{at}f(t)$ | $F(s-a)$ | $f(n-k)u(n-k)$ | $z^{-k}F(z)$ |
| $u(t-a)f(t-a)$ | $e^{-as}F(s)$ | $a^n f(n)$ | $F(\frac{z}{a})$ |

# Formulae - Reliability

## Distribution Theory - Continuous Random Variables

### Mean and Variance of a Continuous Probability Distribution

$$\begin{aligned}\mu &= E(X) = \int_{-\infty}^{\infty} t\ f(t)dt \\ V(X) &= \int_{-\infty}^{\infty} (t-\mu)^2\ f(t)\ dt = E(X^2)\ -\ E(X)^2\end{aligned}$$

### Cumulative Distribution Function

$$F(t) = \int_{-\infty}^{t} xf(x)dx$$

### Reliability Function

$$R(t) = \int_{t}^{\infty} x\ f(x)\ dx = 1 - F(t)$$

### Instantaneous failure rate function

$$\begin{aligned}r(t) &= h(t) = \frac{f(t)}{R(t)} \\ R(t) &= exp\left(-\int_{0}^{t} r(x)\ dx\right)\end{aligned}$$

### Systems

$$R(t)_{\text{series}} = \prod_{k=1}^{n} R_k(t), \qquad R(t)_{\text{parallel}} = 1 - \prod_{k=1}^{n}(1 - R_k(t))$$

### (Negative) Exponential Distribution

$$\begin{aligned}f(t) = \lambda\ e^{-\lambda t}, &\qquad F(t) = 1 - e^{-\lambda t} \\ R(t) = e^{-\lambda t}, &\qquad r(t) = h(t) = \lambda \\ \mu &= \frac{1}{\lambda}\end{aligned}$$

### Wiebull Distribution

$$\begin{aligned}f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} Exp\left(-\left(\frac{t}{\eta}\right)^{\beta}\right), &\qquad F(t) = 1 - Exp\left(-\left(\frac{t}{\eta}\right)^{\beta}\right) \\ R(t) = Exp\left(-\left(\frac{t}{\eta}\right)^{\beta}\right), &\qquad r(t) = h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \\ \mu = \eta\ \Gamma\left(1 + \frac{1}{\beta}\right), &\qquad \Gamma\left(n+1\right) = n!\end{aligned}$$