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CIT Semester 1 Examinations 2018/19

Note to Candidates:	Check the <u>Programme Title</u> and the <u>Module Description</u> to ensure that you have received the correct examination. If in doubt please contact an Invigilator.
Module Title:	Maths Methods and Modelling
Module Code:	MATH8009
Programme Title(s):	HDip Sc Data Science Analytics MSc Data Science & Analytics P1 HDipSc Data Science Analytics PT
Block Code(s):	SDAAN_8_Y5 SDAAN_91Y5 SDAANP8_Y5
External Examiner(s):	Prof. Brien Nolan
Internal Examiner(s):	Dr. Justin Mc Guinness, Dr. Katie O'Brien
Instructions:	Answer all four questions. Show all work and calculations in full.
Duration:	2 Hours
Required Items:	Calculator, Murdoch & Barnes Tables, Log/Formulae Tables

Q1.

- a) Alcohol is metabolised by the body (through enzymes in the liver) in such a way that the blood alcohol content decreases linearly. A recent study by DrinkAware showed that the blood alcohol content (BAC) of an average adult who has consumed four pints of beer (8 standard alcohol units) rises to a maximum of 0.18 g/100mL just after consuming the fourth pint. It was also found that seven hours later the average BAC of the same person is 0.04 g/100mL.
- Derive a linear model to describe the BAC of the average adult after consuming 4 pints of beer, from the time at which maximum BAC is achieved.
 - Draw a graph showing the model, including the two data points used.
 - The drink driving limits in Ireland are 0.05g/100mL for experienced drivers and 0.02g/100mL for learner/novice drivers. Use your model to predict how long both experienced and learner/novice drivers need to wait to drive after drinking four pints of beer.
 - Discuss the validity of the model. Which of your answers in (iii) would you have greater confidence in and why?

[10 marks]

b)

- Describe the difference(s) between a linear model and an exponential model.
- For an exponential model, which is the most influential parameter: the initial value or the growth/decay rate? Explain your answer.

[5 marks]

- c) Paracetamol is administered to a patient and is eliminated from the bloodstream at a rate of 18% per hour.

- If a 100mg dose is administered initially, derive a model to describe the elimination of the drug from the bloodstream.
- What is the half-life of the drug?
- The minimum therapeutic dose required for paracetamol to be effective is 20mg; below this level the drug has no effect. At what time after the initial dose of 100mg is this level reached, i.e. when should a second dose be taken?

[10 marks]

Q2.

- a) A multiple choice exam consists of ten questions, each of which has four possible answers, with only one correct answer. A student who neglected to study for this test decides to trust in chance and randomly chooses their answers. If negative marking is not applied, what is the probability that the student
- i. gets no questions correct?
 - ii. fails the exam (less than four answers correct)?
 - iii. passes the exam?

[8 marks]

- b) The CIT website gets an average of five hits in a ten minute period. What is the probability that the CIT website gets
- i. Exactly 2 hits in the next ten minutes?
 - ii. Less than 3 hits in the next ten minutes?
 - iii. Exactly 40 hits in the next hour?

[8 marks]

- c) A dairy packaging plant fills cartons of milk using an automated machine, however the machine is not perfect and has small differences between each fill. It is found that the carton fills follow a normal distribution with a standard deviation of 0.01L (10mL). To fill 1L cartons, the machine is set to deliver 1.015L in order to avoid under-filling
- i. At this setting, what proportion of cartons will be under-filled?
 - ii. If the 1L cartons are filled with more than 1.05L, spilling occurs which damages the machine and results in a loss of product. At the above setting, what proportion of cartons will be over-filled?
 - iii. What setting much be chosen if no more than 2% of cartons are to be under-filled? What percentage of cartons are overfilled at this setting?

[9 marks]

Q3. A study was carried out to determine if the amount of time spend studying for an exam by students affects their final result. The table below shows the study times per week and the final results for 8 students:

Time studying per week (hours) [x]	10	20	7	14	22	17	24	6
Final grade (percent) [y]	45	70	38	50	65	53	75	35

$$\sum x = 120, \sum y = 431, \sum xy = 7157, \sum x^2 = 2130, \sum y^2 = 24753$$

- a) Using graph paper, plot a scatter diagram for the data. **[4 marks]**
- b) Calculate the coefficient of correlation correct to two decimal places. Comment on your result. **[8 marks]**
- c) Calculate the coefficient of determination correct to two decimal places. Comment on your answer. **[2 marks]**
- d) Use the least squares method to find the regression equation, rounding the values of a and b to two decimal places. Plot the regression line on your scatter diagram from part (a). **[7 marks]**
- e) Using the regression equation you obtained in part (d), estimate the final grade for a student who spent
- i. 3 hours per week studying
 - ii. 18 hours per week studying
- Which of these predictions is more accurate? Why? **[4 marks]**

Correlation Coefficient:

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

Regression Equation: $y = a + bx$, where

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} \qquad a = \frac{\sum y - b \sum x}{n}$$

Q4. Within Institutes of Technology in Ireland, there are three grades of lecturer: Assistant Lecturer (AL), Lecturer (L2), and Senior Lecturer (SL), in order of increasing salary. Yearly reviews for all lecturers take place every January, from which there are three possible outcomes:

- Remain in current position
- Get promoted to a higher position (if one exists)
- Get demoted to a lower position (if one exists)

The probability of getting promoted from AL to L2 is 0.3, while the remainder of Assistant Lecturers remain in that grade. For L2, the probability of being promoted to SL is the same as being demoted to AL, with the probability of remaining in L2 being 0.8. The probability of being demoted from SL to L2 is 0.2 and the remainder of Senior Lecturers remain in their position.

a) Construct a transition diagram to represent this situation, clearly labelling the probabilities.

[4 marks]

b) Construct a transition matrix to represent this situation.

[4 marks]

c) What is the probability that a person who begins as an Assistant Lecturer will be a Senior Lecturer after 3 years of service?

[7 marks]

d) What is the long term probability of achieving and maintaining the position of Senior Lecturer?

[10 marks]